Name: Claire   Date: 11.14   10/10

3.1 Generating TOV , Midpoint, and Linear Functions

1. Indicate which of the following equations are linear functions. If not, explain why.

a) $\frac{3y}{2} = 2x - 1$ ✓	b) $y = \frac{2x-1}{2x}$ ✗	c) $y = 2\sqrt{x} + 4x$ ✗	d) $y = 2x^3 - 3x + 1$ ✗
e) $2y = 2\sqrt{x^2} + 1$ ✓	f) $2y - 1 = 2^x + 1$ ✗	g) $y + 2 = \frac{-2x + 2x^2}{5x}$ ✗	h) $3xy + 1 = 5x$ ✗
i) $\frac{5y}{-2} + 2x = 1$ ✓	j) $\frac{-4}{3}y + 2x - 9$ ✗ not a function.	k) $\frac{3y - 2x}{3} = 4$ ✓	l) $4xy = 3$ ✗

2. Given the following functions, fill in the table of values

a) $y = 3x - 4$

x	−2	0	2	4
y	−10	−4	2	8

b) $-6y = 2x - 1$

x	−2	$-\frac{17}{2}$	2	$-\frac{35}{2}$
y	$\frac{5}{6}$	3	$-\frac{1}{2}$	6

c) $3y = \frac{5x-1}{2}$ −11

x	−2	$-\frac{23}{5}$	10	4
y	$-\frac{11}{6}$	−4	$\frac{49}{6}$	$\frac{19}{6}$

d) $y = \sqrt{2x - 1}$

x	−2		−8	4
y	✗	−3	✗	

e) $y = \frac{1}{2x} + 1$

x	−2	0	$-\frac{1}{10}$	4
y	$\frac{3}{4}$	1	−4	$\frac{9}{8}$

f) $y = 3x^2 - 4$

x	−2	0	2	4
y	8	−4	8	44

3. Given the following table of values, find a function that will satisfy it

a)

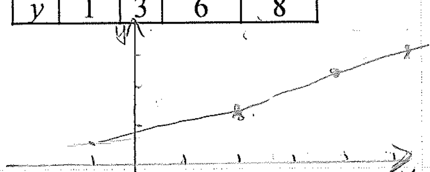
x	0	1	2	3
y	−3	−1	1	3

$y = 2x - 3$

b)

x	−2	3	10.5	15.5
y	1	3	6	8

$y = \frac{2}{5}x + \frac{9}{5}$



c)

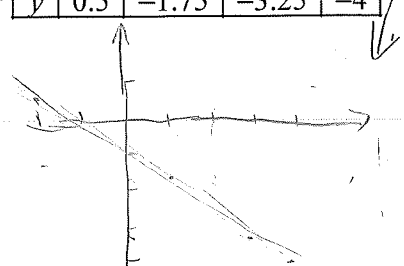
x	1	3	9	11
y	2	3	6	7

$2y - 3 = x$

d)

x	−2	1	3	4
y	0.5	−1.75	−3.25	−4

$y = -\frac{3}{4}x - 1$

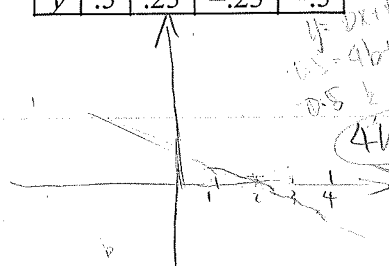


e)

x	0	1	3	4
y	.5	.25	−.25	−.5

$y = bx + b$
$-0.5 = 4b + b$
0.5 b

$4y = -x + 2$



f)

x	15	1	3	4
y	13	−1	1	2

$y = x - 2$

4. Given that the point (3,5) satisfies the equation, what is the value of "k": $3y = 2kx - 10$

$3 \cdot 5 = 2 \cdot 3 \cdot k - 10$

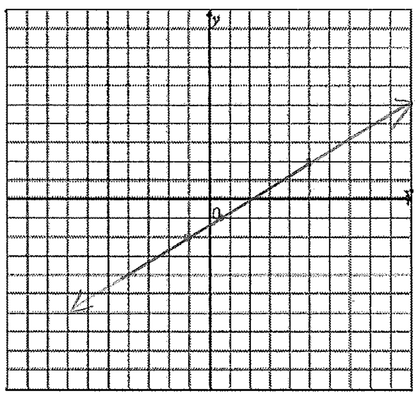
$5 = 6k$

$\boxed{\frac{5}{6}} = k$

5. Graph the following functions on the grid provided

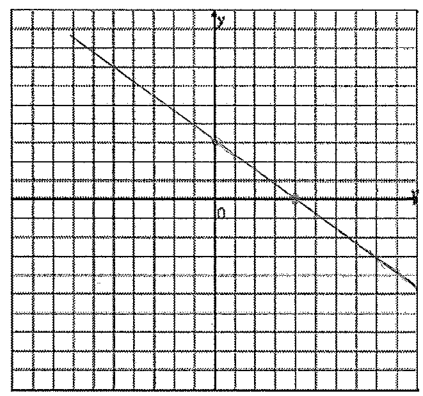
a) $3y = 2x - 4$

x	-1	$\frac{1}{2}$	2	5
y	-2	-1	0	2



b) $3x + 4y = 12$

x	-4	0	4	8
y	6	3	0	-3



6. "A"(-2,4) is an endpoint and "M" (2,-3) is the midpoint of a line segment. Find the coordinate of the other endpoint. $\rightarrow (a,b)$

$\frac{a+(-2)}{2} = 2$ & $\frac{4+b}{2} = -3$

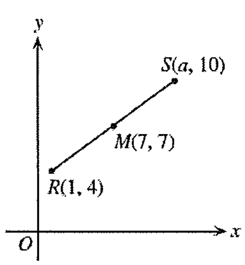
$a = 6$ $b = -10$

$(6, -10)$

7. Find the coordinates of the three points that divide line AB into three equal parts, with coordinates A(8,-12) and B(-4,4)

$AB = 20$ $(-1,0)$ & $(2,-4)$ & $(6,-8)$

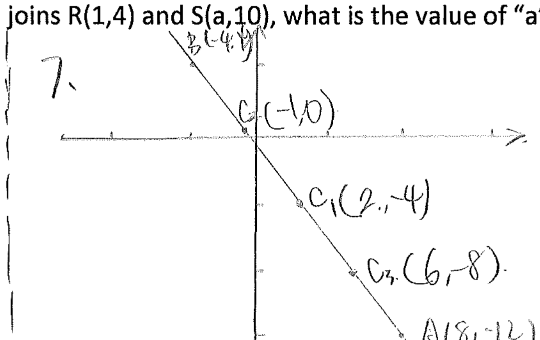
8. If M(7,7) is the midpoint on the line segment which joins R(1,4) and S(a,10), what is the value of "a"?



$\frac{a+1}{2} = 7$

$a = \boxed{13}$

7.



9. The points (x,y) represented in this table lie on a straight line. When the equation of this line is written in the form y=Ax+B, what is the value of A+B?

x	y
2	7
$t-2$	v
t	$v+6$

$7 = 2A + B$ ①

$v = (t-2)A + B$ ②

$v+6 = tA + B$ ③

③ − ②: $6 = 2A$

$3 = A$

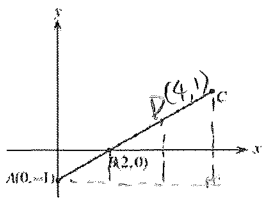
①: $B = 7 - 2A$

$= 7 - 6$

$= 1$

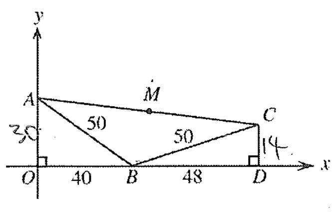
$A + B = 3 + 1 = \boxed{4}$

10. In the diagram, point A,B, and C lie on a line such that *BC=2AB*. What are the coordinates of C?



∵ B is the midpoint of AD.
∴ D(4,1).
∵ D is the midpoint of BC
∴ C(6,2).

11. In the diagram, triangles AOB and CDB are right-angled and "M" is the midpoint of AC. What are the coordinates of M?

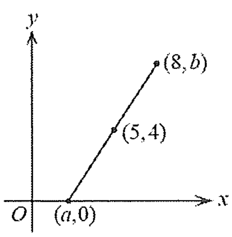


A(0,30), C(88,14).

$\therefore M\left(\frac{0+88}{2}, \frac{30+14}{2}\right)$

$\therefore M(44,22)$.

12. In the diagram, the line segment with endpoints (a,0) and (8,b) has midpoint (5,4). What are the values of "a" and "b"?



$\frac{8+a}{2} = 5$ $\qquad$ $\frac{b+0}{2} = 4$

$8+a = 10$ $\qquad$ $b+0 = 8$

$a = \boxed{2}$ $\qquad$ $b = \boxed{8}$

13. If a line segment $A(x_1, y_1)$ and $B(x_2, y_2)$, find expressions for the coordinates of the points which divide segment AB into i) 3 equal parts and ii) 4 equal parts.

i) 3 equal parts: C_1 and C_2 $\qquad$ ii) 4 equal parts: D_1, D_2 and D_3

A C_1 C_2 B $\quad C_1\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right)$, $C_2\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}\right)$ $D_1\left(\frac{x_1+x_2}{4}, \frac{y_1+y_2}{4}\right)$, $D_2\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, $D_3\left(\frac{3x_1+3x_2}{4}, \frac{3y_1+3y_2}{4}\right)$

14. Given that $0 < x < y < 20$, what is the number of integer solutions (*x*,*y*) to the equation: $2x+3y=50$?

(a) 25 $\qquad$ (b) 16 $\qquad$ (c) 8 $\qquad$ (d) 5 $\qquad$ (e) 3

$y = 2, 4, 6, 8, 10, 12, 14, 16,$

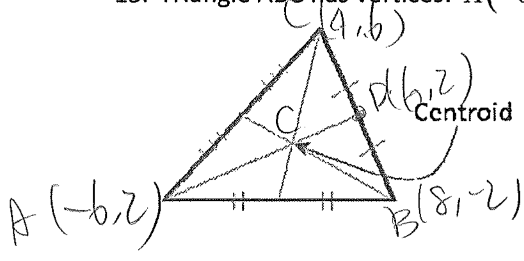
$\boxed{8}$

The number of solutions (x, y) of the equation $3x + y = 100$, where x and y are positive integers, is

(A) 33 $\qquad$ (B) 35 $\qquad$ (C) 100 $\qquad$ (D) 101 $\qquad$ (E) 97

$100 \div 3 = 33R1$.

15. Triangle ABC has vertices: $A(-6,2)$, $B(8,-2)$, and $C(4,6)$. Find the coordinates of its centroid.



$D\left(\frac{4+8}{2}, \frac{6+(-2)}{2}\right)$

$\therefore D(6,2)$

$C\left(\frac{x_1+2x_2}{3}, \frac{y+2y_2}{3}\right)$

$\therefore C\left(\frac{-6+12}{3}, \frac{2+4}{3}\right)$ $\therefore C(2,2)$.

Name: Claire Li Date: 11.18

Math 9 Enriched: Section3.2 Slopes and Lengths of Line Segments.

1. Find the length of each line segment given the coordinates of the endpoint

a) A(3,-5) and B(-4,-9) $AB=\sqrt{(3-(-4))^2+[-5-(-9)]^2}$ $=\sqrt{7^2+4^2}$ $=\sqrt{65}$	b) C(-4,3) and D(7,3) $CD=\sqrt{(-4-7)^2+(3-3)^2}$ $=\sqrt{(11)^2}$ $=11$	c) E(12.5,5) and F(-1.5,5) $EF=12.5-(-1.5)$ $=14$
d) G(3.5, 5) and H(7.5,8.5) $GH=\sqrt{(3.5-7.5)^2+(5-8.5)^2}$ $=\sqrt{4^2+3.5^2}$ $=\sqrt{28.25}=\frac{\sqrt{113}}{2}$	e) J(-3,0) and K(8,-9) $JK=\sqrt{(-3-8)^2+[0-(-9)]^2}$ $=\sqrt{11^2+9^2}$ $=\sqrt{202}$	f) L(-3.83,-11.2) and M(-4.7,-9.4) $LM=\sqrt{(3.83+4.7)^2+(-11.2+9.4)^2}$ $=\sqrt{(8.53)^2+(1.8)^2}$ $=\sqrt{343.3609+3.24}=\sqrt{346.6009}$

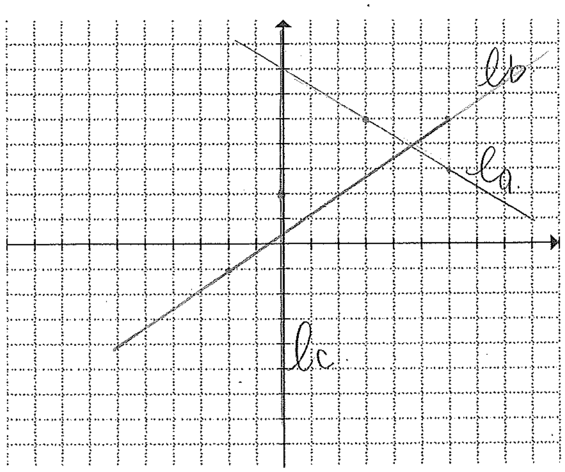
2. Find the slope of each line segment given the coordinates of two points on the line:

a) A(3,-5) and B(-4,-9) $m=\frac{-5+9}{3+4}=\frac{4}{7}$	b) C(3,-5) and D(-4,-9) $m=\frac{-5+9}{3+4}=\frac{4}{7}$	c) A(1,6) and B(5,-4) $m=\frac{6+4}{1-5}=-\frac{10}{4}=-\frac{5}{2}$
d) M(2,-7) and N(-7,2) $m=\frac{-7-2}{2+7}=\frac{-9}{9}=-1$	e) J(3,-5) and K(-4,-5) $m=\frac{-5+5}{3+4}=\frac{0}{7}=0$	f) W(6,-5) and V(6,-9) $m=\frac{-5+9}{(0)}$

3. Given the following coordinates show whether if they are collinear

a) $C(2,5), D(5,8), E(-5,-2)$ $m_{CD}=\frac{5-8}{2-5}=1$. $m_{DE}=\frac{8+2}{5+5}=1$. ∴ they are collinear.	b) $G(-4,8), E(-2,4), F(1,2)$ $m_{GE}=\frac{8-4}{-4+2}=-2$ $m_{EF}=\frac{4-2}{-2-1}=-\frac{2}{3}$ ∴ Not collinear	c) $Y(12,-20), E(5,-11), S(-2,-2.5)$ $m_{YE}=\frac{-20+11}{12-5}=-\frac{9}{7}$ $m_{ES}=\frac{-11-(-2.5)}{5-(-2)}=-\frac{8.5}{7}$ ∴ Not collinear

4. Graph the following lines on the graph:



a) (3,5) & slope = -2/3

(6,3)

$y=-\frac{2}{3}x+b$

$(3,5)$: $5=-2+b$

∴ $b=7$

$y=-\frac{2}{3}x+7$

b) (-2,-1) & slope = 3/4

(6,5)

$y=\frac{3}{4}x+b$

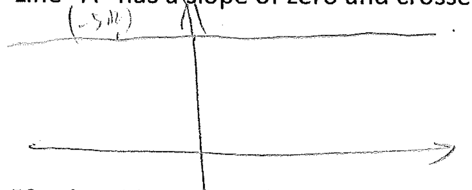
$(-2,-1)$: $-1=-\frac{3}{2}+b$

∴ $b=\frac{1}{2}$

$y=\frac{3}{4}x+\frac{1}{2}$

c) (0,2) & slope is undefined

5. Line "A" has a slope of zero and crosses the point (-5,12). What is the "X" and "Y" intercept of the line?



"y" intercept = (0,12)

There's no "x" intercept.

6. If points 'A", "B" and "C" are on the line: $y = kx + w$, can the slope of segment AB be larger than the slope of segment BC? Justify your answer:

No. AB, BC, AC have the same slope.

7. Find the value of 'k' with the given endpoints and the slope:

a) P(3,9) , Q(-1,3k); slope =-6

$-6 = \frac{9-3k}{3+1}$

$-8 = 3-k$

$k = \boxed{11}$

b) W(-2,-2), X(k,2); slope = $\frac{3}{4}$

$\frac{3}{4} = \frac{-2-2}{-2-k}$

$-16 = -6-3k$

$-10 = -3k$

$\frac{10}{3} = k$

$k = \boxed{\frac{10}{3}}$

c) Y(3,4), Z(9,k); slope= -4/3

$-\frac{4}{3} = \frac{4-k}{3-9}$

$12-3k = 24$

$-3k = 12$

$k = -4$

8. Given that the following line segments are equal, what is the value of "x"?

a) P(-6,-3) , Q(-6,5); R(3,5) S(x,-3)

$PQ = \sqrt{(-6+6)^2+(-3-5)^2}$

$= 8$

$RS = \sqrt{(3-x)^2-(5+3)^2}$

$64 = (3-x)^2-64$

$128 = (3-x)^2$

$8\sqrt{2} = 3-x$

$x = 3-8\sqrt{2}$

b) A(-2x,-1), B(-4,-7), & C(8,1) D(x,7)

$\sqrt{(-2x+4)^2+(-1+7)^2} = \sqrt{(8-x)^2+(1-7)^2}$

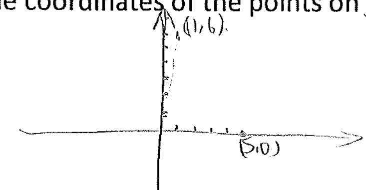
$4x^2-16x+16 = x^2-16x+64$

$4x^2 = x^2+48$

$3x^2 = 48$

$x = 4$

9. Find the coordinates of the points on the y-axis which is equidistant from the points (5,0) and (1,6)



(0,y)

$\sqrt{5+y^2} = \sqrt{1+(y-6)^2}$

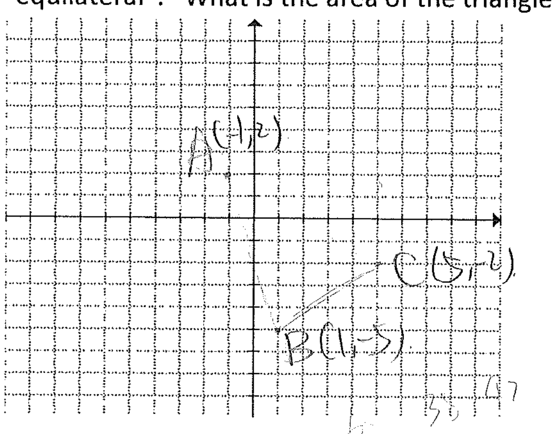
$4+y^2 = y^2-12y+36$

$12y = 32$

$y = \boxed{\frac{8}{3}}$

$(0, \frac{8}{3})$

10. The vertices of a triangle are A(-1,2) , B(1,-5), and C(5,-2). Classify the triangle as "scalene", "isosceles" or "equilateral". What is the area of the triangle?



$AB = \sqrt{(-1-1)^2+(2+5)^2} = \sqrt{4+49} = \sqrt{53}$

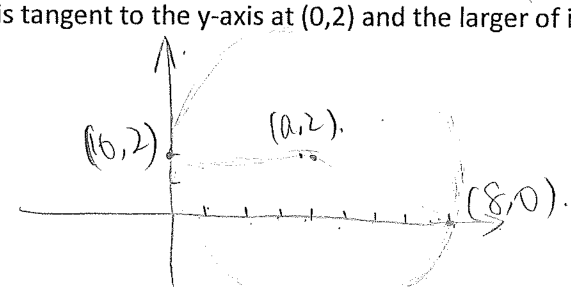
$AC = \sqrt{(-1-5)^2+(2+2)^2} = \sqrt{36+16} = \sqrt{52}$

∴ Scalene triangle

10	-1	2	2
-2	5	-2	-25
5	1	-5	2
13	-1	2	-21

$\frac{13-(-21)}{2} = \boxed{17}$

11. A circle is tangent to the y-axis at (0,2) and the larger of its x-intercept is 8. Determine the radius of the circle.



$\sqrt{(8-a)^2+2^2} = a$

$a^2-16a+64+4 = a^2$

$68 = 16a$

$\boxed{\frac{17}{4}} = a$

$(X, 0)$.

12. Find the coordinates of the points on the x-axis which are 5 units from (5,4).

$\sqrt{(5-x)^2 + 4^2} = 5$ $\quad x = 2$ or 8

$x^2 - 10x + 41 = 25$ $\quad (2,0)$ or $(8,0)$

$x^2 - 10x + 16 = 0$

13. Two line segments, with a common endpoint (0,4) have slopes 2 and -1/2. If the other endpoints are on the x-axis, find their coordinates. $(X_1, 0)$ & $(X_2, 0)$.

$X_1(-2,0)$ & $X_2(-8,0)$.

$-\frac{4}{x_1} = 2$ $\quad -\frac{4}{x_2} = -\frac{1}{2}$

$x_1 = -2$ $\quad 8 = x_2$

14. A line segment has length 10, and its endpoints are on the coordinate axes. If the slope of the line segment is -3/4, find the possible coordinates of the endpoints.

$(-8,6)$. $(0,6)$. $l: y = -\frac{3}{4}x$. $(8,0)$. $(-8,0)$. $(0,-6)$. $(4,3)$.

$\sqrt{x^2 + (\frac{3}{4}x)^2} = 10$

$\sqrt{\frac{25}{16}x^2} = 10$ $\quad x = 8$.

$\frac{5}{4}x = 10$.

$(0,6)$ and $(8,0)$.

$(0,-6)$ and $(-8,0)$

15. Can a triangle be drawn such that all the sides have a positive slope? Explain and justify your answer.

← positive slopes

It is possible.

16. Triangle ABC has vertices A(-4,1), B(2,-1), and C(1,k). What is the number of possible values for "k" such that the triangle is isosceles?

$AB = \sqrt{(-4-2)^2 + (1+1)^2} = \sqrt{36+4} = 2\sqrt{10}$

$1°$ $AB = AC = 2\sqrt{10}$.

$AC = \sqrt{(1+4)^2 + (1-k)^2} = 2\sqrt{10}$ $\quad k = 1 \pm \sqrt{15}$

$2°$ $AB = BC = 2\sqrt{10}$

$BC = \sqrt{(1-2)^2 + (-1-k)^2}$

$2\sqrt{10} = \sqrt{k^2 - 2k + 2}$.

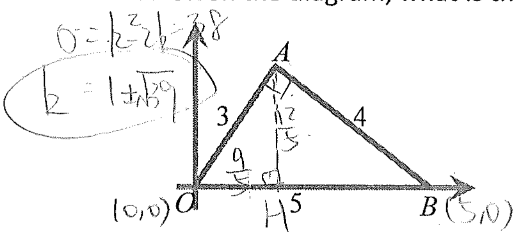
$0 = k^2 - 2k - 38$

$k = 1 \pm \sqrt{39}$

A(-4,1) B(2,-1)

$y = k \times 16 \; 4 + 1 = \boxed{5}$

17. Given the diagram, what is the slope of OA?



$AH = \frac{3 \times 4}{5} = \frac{12}{5}$

$OH = \sqrt{OA^2 - AH^2} = \frac{9}{5}$

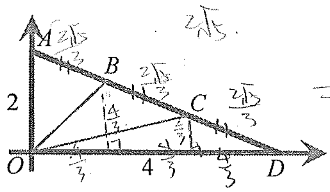
$\therefore A(\frac{9}{5}, \frac{12}{5})$.

l_{OA}: $\frac{12}{5} = \frac{9}{5}k$

$\frac{4}{3} = k$

$\therefore$ the slope of OA is $\frac{4}{3}$

18. If *AB* = *BC* = *CD*, then what is the ratio of the slope of OB to the slope of OC?



$AD = 2\sqrt{5}$

$\Rightarrow B(\frac{4}{3}, \frac{4}{3})$. $C(\frac{8}{3}, \frac{2}{3})$.

The slope of OB $= \frac{4/3}{4/3} = 1$.

The slope of OC $= \frac{2/3}{8/3} = \frac{1}{4}$

$1 : \frac{1}{4}$

$\boxed{4:1}$

19. A line the sum of whose intercepts is "N" will be called an N-line. The sum of the the y-intercepts of the two "3-lines" which pass through the point (-2,-4) is:

(a) −3 (b) −1 (c) 1 (d) 3 (e) 6

20. Given that the following points are collinear A(5,6) B(-4,-3) and $C\left(k, k^2-1\right)$, find all the possible coordinates for C.

$m_{AB} = \frac{-3-6}{-4-5} = \frac{-9}{-9} = 1$

$\therefore l_{AB}: y = x+1$

$f(k) = k^2-1$

$k+1 = k^2-1$

$k^2-k-2=0$

$(k-2)(k+1)=0$

$k=2$ or -1 $\therefore C(-1,0)$ $(2,3)$.

21. Each of the points P(4,1), Q(7,-8), and R(10,1) is on the circumference of circle C. Determine the length of the radius of circle C.

C(x,y).

$PC = \sqrt{(4-x)^2+(1-y)^2}$

$QC = \sqrt{(7-x)^2+(-8-y)^2}$

$RC = \sqrt{(10-x)^2+(1-y)^2}$

$PC = QC$

$(4-x)^2+(1-y)^2 = (7-x)^2+(y+8)^2$

$x^2-8x+16+y^2-2y+1 = x^2-14x+49+y^2+16y+64$

$-8x-2y+17 = -14x+16y+113$

$6x-18y = 96$

22. The point "B" is between the points A(2,3) and C(5,-7) and collinear with "A" and "C". If $AB:BC$ is 3:7, the sum of the coordinates of point "B

(a) $\frac{3}{2}$ (b) $\frac{29}{10}$ (c) $\frac{3}{10}$ (d) $\frac{1}{10}$ (e) 2

$AC = \sqrt{10^2+3^2} = \sqrt{109}$

$\frac{3}{10} \cdot 3 = \frac{9}{10}+2$

$B\left(\frac{29}{10}, 0\right)$

$\frac{29}{10}+0 = \boxed{\frac{29}{10}}$

23. The coordinates of points "A", "B", and "C" are (7,4), (3,1), and (0,k), respectively. The minimum value of $\overline{AB}+\overline{BC}$ is obtained when "k" equals:

$\boxed{3.8}$ // $(k,0)$

(a) 1 (b) 1.7 (c) 1.9 (d) 2.5 (e) 4

$l_{B,A}: y = \frac{5}{4}x - \frac{19}{4}$

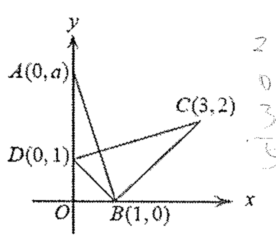
$0 = \frac{5}{4}x - \frac{19}{4}$

$\frac{19}{4} = \frac{5}{4}x$

$\frac{19}{5} = x$

$3.8 = x$

24. In the diagram, A(0,a) lies on the y-axis above "D". If the triangles AOB and BCD have the same area, determine the value of "a". Explain how you got your answer.



$S_{\triangle BCD} = 2 = S_{\triangle AOB}$

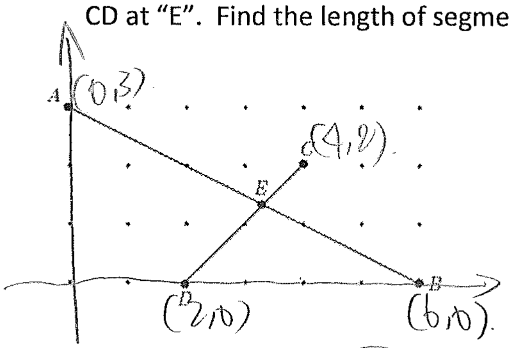
$\therefore 2 = \frac{OD \cdot OB}{2}$

$4 = OA$

$\therefore A(0,4)$

$a = \boxed{4}$ //

25. The diagram shows 28 lattice points, each one unit from its nearest neighbors. Segmenfat AB meets segment CD at "E". Find the length of segment AE.



$l_{CD}: y = x-2$

$l_{AB}: y = -\frac{1}{2}x+3$

$l_{CD} = l_{AB}$

$x-2 = -\frac{1}{2}x+3$

$\therefore E\left(\frac{10}{3}, \frac{4}{3}\right)$

$AE = \sqrt{\left(\frac{10}{3}\right)^2 + \left(3-\frac{4}{3}\right)^2}$

$= \sqrt{\frac{100}{9}+\frac{25}{9}}$

$= \sqrt{\frac{125}{9}}$

$= \frac{5\sqrt{5}}{3}$

(A) $4\sqrt{5}/3$ (B) $5\sqrt{5}/3$ (C) $12\sqrt{5}/7$ (D) $2\sqrt{5}$ (E) $5\sqrt{65}/9$

$\frac{3}{2}x = 5$

$x = \frac{10}{3}$

Name: Claire Li Date: 11.20.

Math 9 Enriched: Section 3.3 Graphing Linear Functions

1. Indicate the slope and y-intercept for each of the following equations:

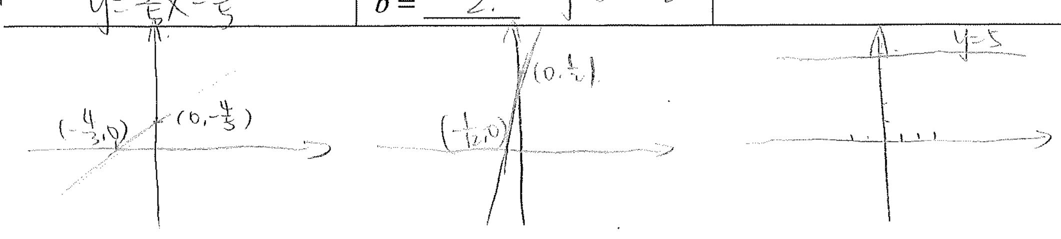
a) $y=3x-4$ $m=$ 3 $b=$ -4	b) $y=4+2x$ $m=$ 2 $b=$ 4	c) $3y=-5x+2$ $y=-\frac{5}{3}x+\frac{2}{3}$ $m=$ $-\frac{5}{3}$ $b=$ $\frac{2}{3}$	d) $y=\frac{3x-4}{3-2}$ $m=$ $-\frac{3}{2}$ $b=$ 2
e) $4y-5x=-12$ $4y=5x-12$. $y=\frac{5}{4}x-3$. $m=$ $\frac{5}{4}$ $b=$ -3	f) $5y=-4$ $y=-\frac{4}{5}$. $m=$ 0. $b=$ $-\frac{4}{5}$	g) $3x=-4y+2$ $y=\frac{1}{2}-\frac{3}{4}x$. $m=$ $\frac{-3}{4}$ $b=$ $\frac{1}{2}$	h) $x+y=3x-4y$ $5y=2x$. $y=\frac{2}{5}x$. $m=$ $\frac{2}{5}$ $b=$ 0.
i) $-\frac{y}{2}=\frac{-2x-4}{3}$ $y=\frac{4x}{3}+\frac{8}{3}$. $m=$ $\frac{4}{3}$ $b=$ $\frac{8}{3}$	j) $4y=2(x-4+2y)$ $4y=2x-8+4y$ $8y=2x-8$ $y=\frac{1}{4}x-1$ $m=$ $\frac{1}{4}$ $b=$ -1	k) $3y=3(3x+2y)$ $3y=9x+6y$ $-3y=9x$ $y=-3x$ $m=$ -3 $b=$ 0	l) $20=3x-15$ $m=$ ____ $b=$ ____

2. Given each pair of coordinates, find the slope, y-intercept, and equation of the line:

a) $(-2,3)\&(4,5)$ $m=\frac{5-3}{4+2}=\frac{2}{6}=\frac{1}{3}$ $m=$ $\frac{1}{3}$ $y=\frac{1}{3}x+\frac{11}{3}$ $b=$ $\frac{11}{3}$ $y=$ $\frac{1}{3}x+\frac{11}{3}$.	b) $(-4,6)\&(6,-4)$ $\frac{6+4}{-4-6}=-\frac{10}{10}=-1$ $m=$ -1 $y=-x+b$ $b=$ 2 $-4=-6+b$. $b=2$. $y=$ $x+2$	c) $(-3,5)\&(-2,5)$ $\frac{5-5}{-3+2}=\frac{0}{-1}=0$. $m=$ 0. $y=5$ $b=$ 5 $y=$ 5	d) $(a+2,b)\&(a-2,3b)$ $\frac{b-3b}{a+2-a+2}=\frac{-2b}{4}=-\frac{b}{2}$. $m=$ $-\frac{b}{2}$. $b=$ $2b+ab+2$. $y=-\frac{b}{2}x+B$. $b=-\frac{b}{2}(a+2)+B$. $2b=-ab+2+B$. $B=2b+ab+2$. $y=$ $-\frac{b}{2}x+2b+ab+2$.
e) $(2a,-a)\&(-a+1,2a)$ $\frac{2a+a}{-a+1-2a}=\frac{3a}{-3a+1}$ $m=$ $\frac{3a}{-3a+1}$ $b=$ $-a$. $y=$ $\frac{3a}{1-3a}x-a$	f) $(1,1)\&(a,a^2)$ $\frac{a^2-1}{a-1}$. $\frac{(a-1)(a+1)}{a-1}=a+1$ $m=$ $a+1$ $b=$ $-a$. $1=(a+1)1+b$ $-a$ b. $y=$ $(a+1)x-a$.	g) $(2a+1,a)\&(a,2a+1)$ $\frac{2a+1-a}{a-2a+1}=\frac{a+1}{-a+1}$ $m=$ $\frac{1+a}{1-a}$. $b=$ $\frac{1-3a^2}{1-a}$ $y=$ $\frac{1+a}{1-a}x-\frac{1-3a^2}{1-a}$	h) $(4a^2,9)\&(36,a^2)$ $m=\frac{a^2-9}{36-4a^2}=\frac{(a^2-9)}{-4(a^2-9)}=\frac{1}{4}$. $m=$ $\frac{1}{4}$. $b=$ $9-a^2$ $y=\frac{1}{4}x+b$ $9=\frac{1}{4}\cdot 4a^2+b$ $9=a^2+b$. $y=$ $\frac{1}{4}x+9-a^2$. $b=9-a^2$.

3. Find the "x" and "y" intercepts, and graph the line:

a) $3x+5y=-4$ x int: $(\frac{4}{3},0)$ y int: $(0,-\frac{4}{5})$ $m=$ $\frac{3}{5}$. $b=$ $-\frac{4}{5}$. $y=\frac{3}{5}x-\frac{4}{5}$	b) $4x=\frac{2y-1}{3}$ x int: $(\frac{3}{12},0)$ y int: $(0,\frac{1}{2})$ $m=$ 6 $b=$ $\frac{1}{2}$. $y=6x+\frac{1}{2}$	c) $(-3,5)\&(-2,5)$ $y=5$. $m=$ 0 $b=$ 5	d) $(a+2,b)\&(a-2,3b)$ $\frac{3b-b}{a-2-a-2}=\frac{2b}{-4}=-\frac{b}{2}$. $m=$ $-\frac{b}{2}$ $b=$ $\frac{ab+4b}{2}$.



4. Graph the following lines on the grid to the right:

i) $y=\frac{2x-1}{2}$

ii) $2x+y=4$

X int = (2, 0)

y int = (0, 4).

iii) $x-2y-8=0$

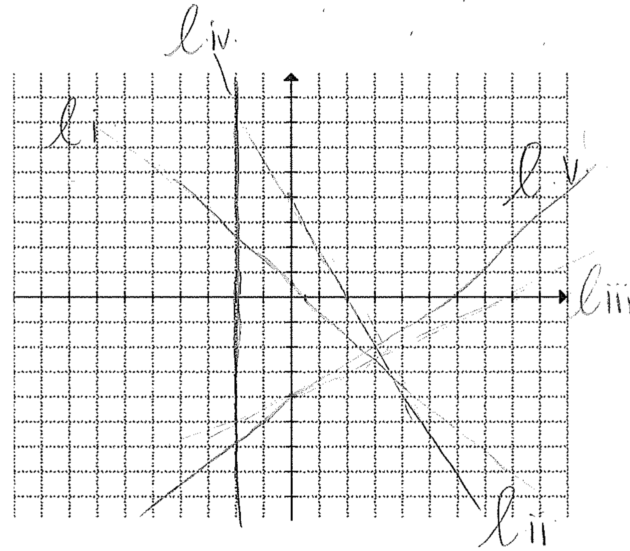
X int = (8, 0)

y int = (0, -4).

iv) $3x+6=0$

v) $3y=2x-12$

allen
alisa
sophie
kit
shan



5. Indicate which of the following lines have the same x-intercept? Y-intercept? Slope?

a) $x-y+2=0$

X int: (-2, 0).

y int: (0, 2).

m = 1

b) $3x+6y+5=0$

X int: $(-\frac{5}{3}, 0)$

y int: $(0, -\frac{5}{6})$

$m=-\frac{1}{2}$.

c) $2x-y-8=0$

X int: (4, 0)

y int: (0, -8)

m = 2

d) $2x+4y-9=0$

X int: $(\frac{9}{2}, 0)$

y int: $(0, \frac{9}{4})$

$m: -\frac{1}{2}$

e) $4x-3y=-6$

X int: $(-\frac{3}{2}, 0)$

y int: (0, 2)

$m=\frac{4}{3}$

X-intercept: / Y-intercept: a & e. Slope b & d

6. If $px+2y=7$ and $3x+qy=5$ represent the same straight line, then what does "p-q" equal?

X int: $(\frac{7}{p}, 0) = (\frac{5}{3}, 0)$

$\frac{7}{p}=\frac{5}{3}$

$p=\frac{21}{5}$.

y int = $(0, \frac{7}{2}) = (0, \frac{5}{q})$

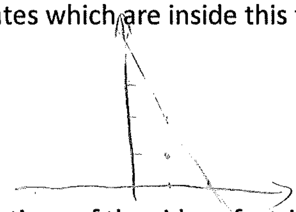
$\frac{7}{2}=\frac{5}{q}$

$q=\frac{10}{7}$

$p-q=\frac{21}{5}-\frac{10}{7}$

$=\frac{147-50}{35}=\frac{97}{35}$

7. The lines $x=0$, $y=0$, and $2x+y=4$ form a triangle. What is the number of points with integral coordinates which are inside this triangle?



1 point.

(1, 1)

8. The equations of the sides of a triangle are $\overset{l_1}{2x+3y=18}, \overset{l_2}{5x+y+7=0}, and \overset{l_3}{3x-2y=14}$

a) Graph the lines on the same grid and find the vertices

b) Find the slopes of the sides

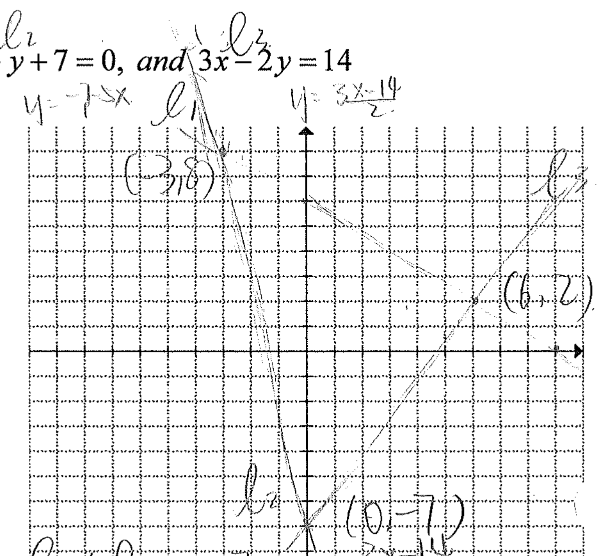
c) Find the area of the triangle

$y=\frac{18-2x}{3}$.

$y=-7-5x$

$y=\frac{3x-14}{2}$

$\boxed{\frac{117}{2}}$



-3 8
18 -6 2 -6
0 -0 -7 -42
21 -3 8 -0
69 -48

$\frac{69-(-48)}{2}$

$=\frac{117}{2}$

l_1+l_2: $\frac{18-2x}{3}=-7-5x$

$18-2x=-21-15x$

$13x=-39$

$x=-3$.

(-3, 8).

l_1+l_3: $\frac{18-2x}{3}=\frac{3x-14}{2}$

$36-4x=9x-42$

$78=13x$.

$x=6$

(6, 2).

l_2+l_3: $-7-5x=\frac{3x-14}{2}$.

$-14-10x=3x-14$

$0=13x$ $x=0$.

(0, -7).

9. Triangle ABC has vertices A(-3,-1), B(1,1), and C(5,-5). The median is drawn from point A to the midpoint of side BC. Determine the equation of the line containing this median.

midpoint of BC $D\left(\frac{1+5}{2}, \frac{1+(-5)}{2}\right) \Rightarrow D(3,-2)$.

$m_{AD} = \frac{-2+1}{3+3} = \frac{-1}{6}$: l_{AB} $y = -\frac{1}{6}x - \frac{3}{2}$

10. Statistics Canada projects that the percent of the Canadian population who will be atleast 65 years old will be 14% in 2011 and 17.5% in the year 2021. If the rate of increase stays the same for the next 20 years, then predict the percentage of the population that will be atleast 65 in the year 2031.

17.5% − 14% = 3.5%

17.5% + 3.5% = $\boxed{21\%}$

11. Given that an linear equation in standard form is: $Ax + By + C = 0$, describe the graph when:

$a) A = 0, B \neq 0, C \neq 0$ $\quad b) B = 0, A \neq 0, C \neq 0$ $\quad c) C = 0, A \neq 0, B \neq 0$

The line is horizontal. The line is vertical. The line passes the origin.

$d) A = 0, C = 0, B \neq 0$ $\quad e) B = 0, C = 0, A \neq 0$

$y = 0$. $\quad x = 0$.

12. Given that 0 < x < y < 20, what is the number of integer solutions (x,y) to the equation 2x + 3y = 50 :

y is even.

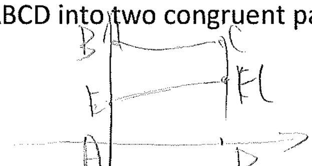
$2x = 50 - 3y$

$x = \frac{50-3y}{2}$

$y = \underbrace{0, 2, 4, 6, 8, 10}_{x \geq 20}, 12, 14, 16, 18$.

4 solutions.

13. A square has vertices A(0,0) , B (0,5), C(5,5), and D (5,0). A line passing through the point E(0,2) divides the square ABCD into two congruent parts. Determine the equation of the line: $m_{EF} = \frac{2-3}{0-5} = \frac{-1}{-5} = \frac{1}{5}$



FD = BE = 5 − 2 = 3.

∴ F(5,3)

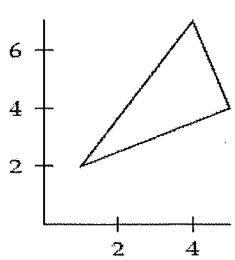
l_{EF}: $y = \frac{1}{5}x + 2$

14. 12. Using the standard xy-coördinate plane, what is the area (in square units) of a triangle whose vertices have the coordinates (0,0) , (1,5), and (7,3)?

	0	0	
0	1	5	0
35	7	3	3
0	0	0	0

$\frac{35-3}{2} = \frac{32}{2} = \boxed{16}$

15. 13. What is the area of the triangle with vertices at the point with coordinates (1,2), (5,4), and (4,7)?



	1	2	
10	5	4	4
16	4	7	35
7	1	2	8
33			47

$\frac{47-33}{2} = \frac{14}{2} = \boxed{7}$

16. In the magic square shown, the sums of the numbers in each row, column, and diagonal are the same. Five of these numbers are represented by *v, w, x, y, and z*. Find the value of *y + z*

v	*24*	19
18	22	*y*
25	*z*	*21*

$18+25+x = v+24+w$
$19 = w$
$18+25+x = v+x+21$
$22 = x$
$25+22+19 = 66$

$25+z+21=66$
$46+z=66$
$z=20$

$19+y+21=66$
$40+y=66$
$y=26$

$y+z = 26+20 = \boxed{46}$

17. If $a+1=b+2=c+3+d+4=a+b+c+d+5$, then what is the value of $a+b+c+d$?

$a+3+d+4 = a+b+c+d+5$
$2 = a+b$ ①

$a+1=b+2$
$a=b+1$ ②

①&②: $2=b+1+b$
$1=2b$
$b=\frac{1}{2}$

$b+2 = a+b+c+d+5$
$\frac{1}{2}+2 = a+b+c+d+5$
$\boxed{-\frac{5}{2}} = a+b+c+d$

18. Given that the constants "A", "B", and "C" are all positive integers, indicate which graph best illustrates the following functions:

A i) *Ax – By = C*
$By = Ax - C$
$y = \frac{A}{B}x - \frac{C}{B}$

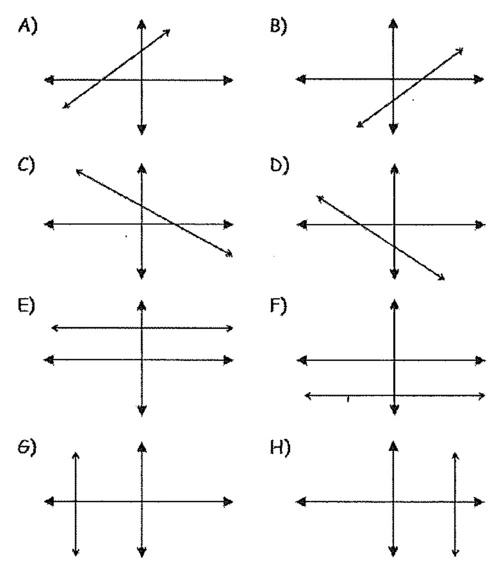
D ii) *– By = C + Ax*
$By = -C - Ax$
$y = -\frac{A}{B}x - \frac{C}{B}$

H iii) *Ax – B = C*
$Ax = B + C$
$x = \frac{B+C}{A}$

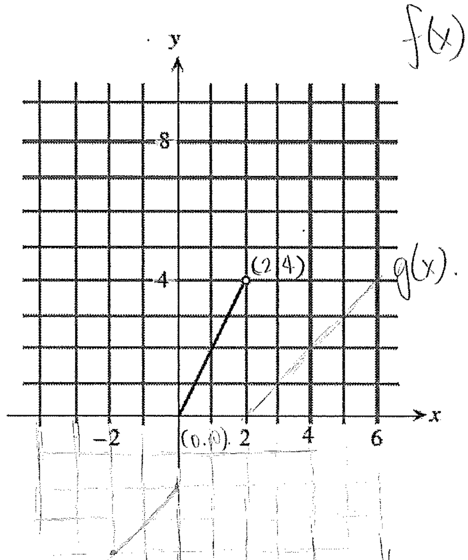
F iv) *A + By =–C*
$By = -C - A$
$y = -\frac{C+A}{B}$

C v) *Ax + By = C*
$By = -Ax + C$
$y = -\frac{A}{B}x + \frac{C}{B}$
$m = -\frac{A}{B}$
$b = \frac{C}{B}$

D vi) *Ax + By = –C*
$By = -Ax - C$
$y = -\frac{A}{B}x - \frac{C}{B}$
$m = -\frac{A}{B}$
$b = -\frac{C}{B}$



19. Part of the graph for $y = f(x)$ is shown, $0 \le x < 2$. If $g(x+2) = \frac{1}{2}f(x)$ for all real values of "x", draw the graph of g(x) for the intervals, $-2 \le x < 0$ and $2 \le x < 6$



$f(x) = y = 2x$
$g(x+2) = \frac{1}{2}f(x)$
$g(x+2) = \frac{1}{2} \cdot 2x$
$g(x+2) = x$
$g(x) = x-2$

Name: Claire Li Date: 12.3.

Section 3.4 Slopes of Parallel and Perpendicular Lines

1. Given the slopes for each line, indicate whether if they are perpendicular, parallel or neither:

a) $m_1 = \frac{2}{3}$, $m_2 = -\frac{3}{2}$

perpendicular

$m_1 \times m_2 = \frac{2}{3} \cdot (-\frac{3}{2}) = -1$

$\therefore m_1 \perp m_2$

b) $m_1 = -\frac{3}{5}$, $m_2 = -\frac{6}{10}$

parallel

$m_1 = m_2 = -\frac{3}{5}$

$\therefore m_1 // m_2$

c) $m_1 = 2$, $m_2 = -\frac{1}{2}$

perpendicular

$m_1 \cdot m_2 = 2 \cdot -\frac{1}{2} = -1$

$\therefore m_1 \perp m_2$

d) $m_1 = \frac{4}{7}$, $m_2 = -\frac{4}{7}$

neither.

e) $m_1 = \frac{12}{14}$, $m_2 = -\frac{7}{6}$

perpendicular.

$m_1 \cdot m_2 = \frac{12}{14} \cdot -\frac{7}{6} = -1$

$\therefore m_1 \perp m_2$

f) $m_1 = 0$, $m_2 = 9999$

neither

g) $m_1 = 0.125$, $m_2 = -8$

Perpendicular

$= \frac{1}{8}$

$m_1 \cdot m_2 = \frac{1}{8} \cdot -8 = -1$

$\therefore m_1 \perp m_2$

h) $m_1 = -0.\overline{18}$ $m_2 = 5.5$

perpendicular

$= -\frac{2}{11}$ $= \frac{11}{2}$

$m_1 \cdot m_2 = -\frac{2}{11} \cdot \frac{11}{2} = -1$

$\therefore m_1 \perp m_2$

2. Given that the two slopes are perpendicular, find the value of *"k"*

a) $m_1 = \frac{4}{6}$, $m_2 = -\frac{1}{k}$

$m_1 \cdot m_2 = -1$

$\frac{4}{6} \cdot -\frac{1}{k} = -1$

$\frac{4}{6k} = 1$

$k = \boxed{\frac{2}{3}}$

b) $m_1 = -\frac{1.5}{4}$, $m_2 = -k+1$

$-\frac{1.5}{4} \cdot (-k+1) = -1$ $k = -\frac{5}{3}$

$\frac{3}{8}(1-k) = 1$

$3 - 3k = 8$

c) $m_1 = \frac{7}{3}$, $m_2 = -\frac{3}{2k}$

$m_1 \cdot m_2 = -1$

$\frac{7}{3} \cdot \frac{3}{2k} = 1$

$7 = 2k$

$\frac{7}{2} = k$

d) $m_1 = -\frac{2}{4.5}$, $m_2 = k - 1$

$-\frac{2}{4.5}(k-1) = -1$

$4k - 4 = 9$

$4k = 13$

$k = \frac{13}{4}$

e) $m_1 = 0.\overline{09}$, $m_2 = k(10-k)$

$0.\overline{09} k(10-k) = -1$

$k(10-k) = -11$

$10k - k^2 = -11$

$k^2 - 10k - 11 = 0$

$(k-11)(k+1) = 0$

$k = 11$ or -1

f) $m_1 = -0.\overline{1}$, $m_2 = 4k^2$

$-0.\overline{1} \cdot 4k^2 = -1$

$\frac{4}{9}k^2 = 1$

$(\frac{2}{3}k)^2 = 1$

$\frac{2}{3}k = \pm 1$ $k = \pm\frac{3}{2}$

g) $m_1 = \frac{-3k}{2}$, $m_2 = \frac{k-1}{7k-6}$

$\frac{-3k}{2} \cdot \frac{k-1}{7k-6} = -1$

$\frac{k-1}{7k-6} = \frac{2}{3k}$

$14k - 12 = 3k^2 - 3k$

$3k^2 - 17k + 12 = 0$

h) $m_1 = -0.\overline{18}$ $m_2 = 3k^2 - 2k$

$-0.\overline{18}(3k^2 - 2k) = -1$

$\frac{2}{11}(3k^2 - 2k) = 1$

$6k^2 - 4k = 11$

$6k^2 - 4k - 11 = 0$

$k = \frac{4 \pm \sqrt{16 + 264}}{12}$

$= \frac{2 \pm \sqrt{70}}{6}$

3. Indicate whether if the following lines segments are either "parallel", "perpendicular" or "neither"

a) M(-5,1) and N(1,-5)
O(1,4) and P(6,-1)

Parallel.

$m_{MN} = \frac{-5-1}{1+5} = \frac{-6}{6} = -1$

$m_{OP} = \frac{4+1}{1-6} = \frac{5}{-5} = -1$

$\therefore l_{MN} // l_{OP}$

b) R(-1,4) and S(3,-2)
T(4,2) and U(-2,-2)

Perpendicular.

$m_{RS} = \frac{4+2}{-1-3} = \frac{6}{-4} = -\frac{3}{2}$

$m_{TU} = \frac{2+2}{4+2} = \frac{4}{6} = \frac{2}{3}$

$-\frac{3}{2} \cdot \frac{2}{3} = -1$ $\therefore l_{RS} \perp l_{TU}$

c) A(-1,3) and B(4,-1)
D(4,5) and C(-2,-3)

$m_{AB} = \frac{3+1}{-1-4} = \frac{4}{-5}$

$m_{CD} = \frac{-3-5}{-2-4} = \frac{8}{6} = \frac{4}{3}$

neither

d) W(-2,3) and V(8,3)
B(4,8) and D(4,-4)

$m_{WV} = \frac{3-3}{8+2} = 0$

$m_{BD} = \frac{8+4}{4-4} =$

Perpendicular.

e) D(3,5) and E(0,-2)
G(5,3) and F(3,-2)

$m_{DE} = \frac{5+2}{3-0} = \frac{7}{3}$

$m_{GF} = \frac{3+2}{5-3} = \frac{5}{2}$

Neither.

f) W(5,4) and V(-9,4)
Y(6,-10) and Z(6,-2)

l_{WV} is horizontal

l_{YZ} is vertical

Perpendicular.

4. Given each pair of coordinates, find the equation of the perpendicular bisector and draw it on the graph provided.

$\ell a \perp \ell_{AB}$ $\ell b \perp \ell_{CD}$ $\ell c \perp \ell_{EF}$

a) $A(-3,4)B(3,5)$

Midpoint of AB: $\left(\frac{3-3}{2}, \frac{5+4}{2}\right)$ $\left(0, \frac{9}{2}\right)$

$m_{AB} = \frac{5-4}{3+3} = \frac{1}{6}$

$\therefore m_a = -1/\frac{1}{6} = -6$

$\therefore \ell a: y = -6x + \frac{9}{2}$

b) $C(-5,-4)D(3,-6)$

Midpoint of CD: $\left(\frac{-5+3}{2}, \frac{-4-6}{2}\right)$ $(-1,-5)$

$m_{CD} = \frac{-6+4}{3+5} = \frac{-2}{8} = -\frac{1}{4}$

$m_b = -1/-\frac{1}{4} = 4$

$\therefore \ell b: y = 4x - 1$

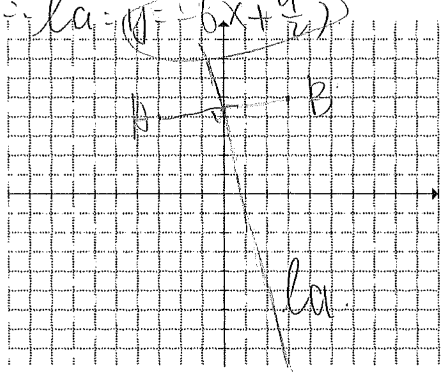
c) $E(9,11)F(1,1)$

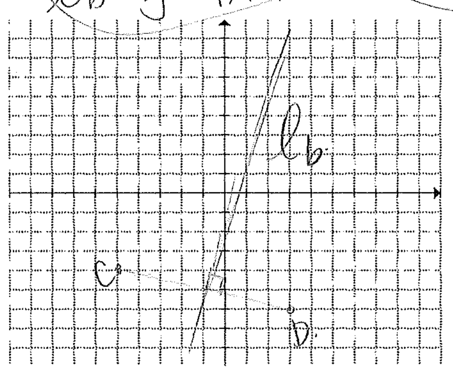
Midpoint of EF: $\left(\frac{9+1}{2}, \frac{11+1}{2}\right)$ $(5,6)$

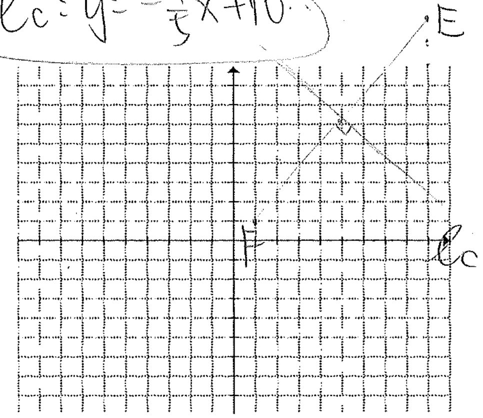
$m_{EF} = \frac{11-1}{9-1} = \frac{10}{8} = \frac{5}{4}$

$m_c = -1/\frac{5}{4} = -1 \cdot \frac{4}{5} = -\frac{4}{5}$

$\therefore \ell c: y = -\frac{4}{5}x + 10$







5. The line $y = ax + c$ is parallel to the line $y = 2x$ and passes through the point (1,5). What is the value of "c"?

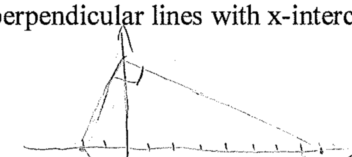
$\because$ They are parallel. $\therefore a = 2$.

$y = 2x + C$

$5 = 2 + C$

$\boxed{3} = C$

6. Two perpendicular lines with x-intercepts -2 and 8 intersect at (0,b). Determine all values of "b"



$\begin{cases} y = ax + b \rightarrow 0 = -2a + b \\ y = -\frac{1}{a}x + b \rightarrow 0 = -\frac{8}{a} + b \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = 4 \end{cases}$ or $\begin{cases} a = -2 \\ b = -4 \end{cases}$ $b = \boxed{\pm 4}$

7. Two perpendicular lines intersect at the point (9,2). If the x-intercept of one line is double the x-intercept of the other, then a possible sum of the x-intercepts is:

$(a,0)$ $(2a,0)$

a) $\frac{17}{2}$	b) 10	c) $\frac{51}{2}$	d) $\frac{45}{2}$	e) 5

$\frac{2}{9-a} \cdot \frac{2}{9-2a} = -1$

$(9-a)(2a-9) = 4$

$18a - 81 - 2a^2 + 9a = 4$

$-2a^2 + 27a - 85 = 0$

$2a^2 - 27a + 85 = 0$

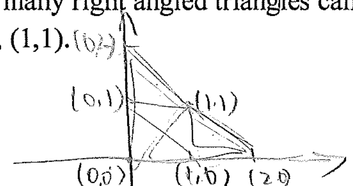
$(a-5)(2a-17) = 0$

$a = 5$ or $\frac{17}{2}$

$\frac{17}{2} + 2 \cdot \frac{17}{2} = \frac{51}{2}$

or $5 + 2 \times 5 = 15$

8. How many right angled triangles can be formed by using any three of the six points: (0,0) , (0,1) , (0,2) , (1,0) , (2,0), (1,1).



$4 + 2 + 1 = \boxed{7}$

9. Two perpendicular lines L_1 and L_2 intersect at the point Q$(p,2p)$. If $(p-6, p)$ is on L_1 and $(p+4,-p)$ is on L_2. Which of the following statement will be true?

a) Q may be any point on the line y=2x

b) there are exactly 3 positions for Q

c) There is exactly one position for Q

d) There are exactly 2 positions for Q

e) The number of possible positions for Q is finite but more than three

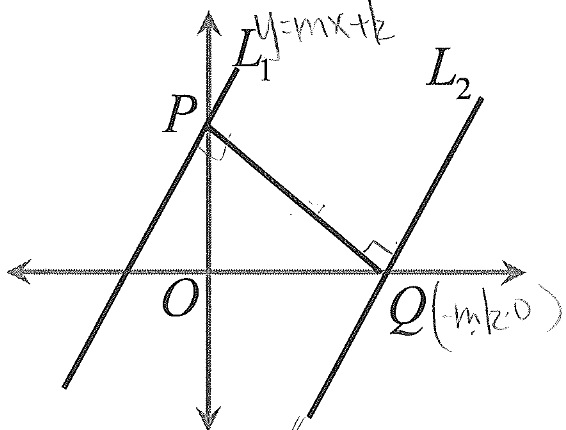
$\frac{p}{2 \cdot 6} \cdot \frac{3p}{-4} = -1$

$\frac{p^2}{-8} = -1$

$p^2 = 8$

$p = 2\sqrt{2}$.

10. The line L_1 has equation $y = mx + k$. Line L_1 crosses the y-axis at "P" and line L_2 crosses the x-axis at Q. If PQ is perpendicular to both lines, then what is the y-intercept of line L_2 $(0, b)$.



$l_1 : y = mx + k$

$l_2 : y = mx + b$

$l_{PQ} : y = -\frac{1}{m}x + k$

$0 = -\frac{1}{m}x + k$

$-mk = x$

$\therefore Q(-mk, 0)$

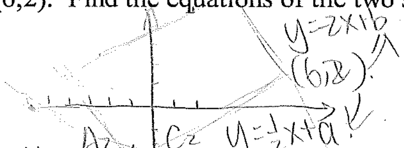
$0 = -m^2k + b$

$b = m^2k$

$\boxed{(0, m^2k)}$ //

11. The equation of two sides of a rectangle are $x - 2y + 8 = 0$ and $2x + y + 6 = 0$. The vertex, which does not lie on either of these lines is $A(6,2)$. Find the equations of the two sides passing through "A".

$\begin{cases} y = \frac{1}{2}x + 4 \\ y = -2x - 6 \end{cases}$



$2 = -12 + b$

$14 = b$

$b = 14$

$a = 5$

$\begin{cases} y = -2x + 14 \\ y = \frac{1}{2}x + 5 \end{cases}$

$y = -\frac{A_1}{B_1}x - \frac{C_1}{B_1}$ $\quad$ $y = -\frac{A_2}{B_2}x - \frac{C_2}{B_2}$

12. $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ are two different lines. What relations exist among the values of $A_1,\ B_1,\ C_1,\ A_2,\ B_2,\ and\ C_2$ if the lines are:

a) Parallel (but do not overlap)

$\frac{A_1}{B_1} = \frac{A_2}{B_2}$

$A_2B_1 = A_1B_2$

b) perpendicular

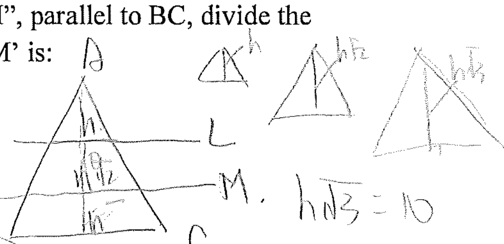
$-\frac{A_1}{B_1} = \frac{B_2}{A_2}$

$B_1B_2 = -A_1A_2$

13. The distance from point A to the line segment BC is 10cm. Two lines "L" and "M", parallel to BC, divide the triangle ABC into three parts of equal area. The distance between lines "L" and "M' is:

(A) $10\left(1 - \frac{\sqrt{3}}{3}\right)$ (B) $10\frac{\sqrt{3}}{3}$ (C) $10\frac{\sqrt{6}}{3}$

(D) $10\frac{\sqrt{3}}{3}\left(\sqrt{2} - 1\right)$ (E) $10\left(1 - \frac{\sqrt{6}}{3}\right)$



$h\sqrt{3} = 10$

$h = \frac{10\sqrt{3}}{3}$

$(\sqrt{2} - 1) \cdot \frac{10\sqrt{3}}{3}$

14. Suppose the line "L" is parallel to the line $y = \frac{3}{4}x + 6$ and four units it. Which of the following is a possible equation of line "L":

(a) $y = \frac{3}{4}x$ (b) $y = \frac{3}{4}x + 1$ (c) $y = \frac{3}{4}x + 2$ (d) $y = \frac{3}{4}x + 3$ (e) $y = \frac{3}{4}x + \frac{9}{2}$

$10\sqrt{3}$

15. Three straight lines L1, L2, and L3 have slopes $\frac{1}{2}, \frac{1}{3},\ and\ \frac{1}{4}$ respectively. All three lines have the same y-intercept. The sum of the x-intercepts of the lines is 36. Determine the y-intercept.

$\begin{cases} l_1 : y = \frac{1}{2}x + b \rightarrow x\text{-int} = -2b \\ l_2 : y = \frac{1}{3}x + b \rightarrow x\text{-int} = -3b \\ l_3 : y = \frac{1}{4}x + b \rightarrow x\text{-int} = -4b \end{cases}$

$-2b - 3b - 4b = 36$

$-9b = 36$

$b = -4$

$\boxed{(0, -4)}$ //

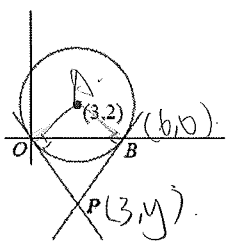
16. If "a", "b", "c" are real numbers, what is the only value of "a" for which the graphs of $y = ax + b$ and $x = ay + c$ are perpendicular to each other?

$y = ax + b$. $y = \frac{1}{a}x - \frac{c}{a}$.

$\boxed{a = 0}$

17. A circle with center at (3,2) intersects the x-axis at the origin, O, and at the point "B". The tangents to the circle at "O" and "B" intersect at point "P". The y-coordinates of "P" is:

a) -3.5 b) -4 c) -4.5 d) -5 e) None of these



$m_{AB} \times m_{BP} = -1$

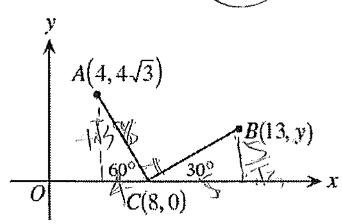
$\frac{2}{-3} \cdot -\frac{y}{3} = -1$

$2y = -9$

$y = -\frac{9}{2}$

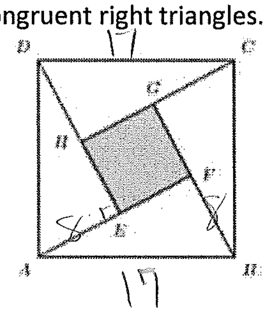
18. In the diagram, the value of "y" is:

a) $\frac{13}{2\sqrt{3}}$ b) $\frac{5}{\sqrt{3}}$ c) 2 d) 12 e) $\frac{\sqrt{3}}{5}$



$\frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

19. In the diagram, $ABCD$ is a square with side length 17 and the four triangles $ABF, ADE, BCG,$ and CDH are congruent right triangles. Further, $\overline{FB}$ =8. The area of the shaded quadrilateral $EFGH$ is



(A) $12\sqrt{17}$ (B) 49 (C) $\frac{169}{4}$

(D) 61 (E) 81

$AF = \sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15$

$(15 - 8)^2 = 7^2 = 49$

20. Two perpendicualr lines L_1 and L_2 intersect at the point $Q(p, 2p)$ in the first quadrant. If $S(p-6, p)$ is on L_1 and $T(p+6, -p)$ is on L_2, then:

$\frac{2p-p}{p-p+6} \cdot \frac{3p}{-6} = -1$

(a) Q may be any point on the line $y = 2x$

(b) there is no such point Q

$\frac{3p^2}{36} = 1$

(c) there is exactly one possible position for the point Q

(d) there are exactly two possible positions for the point Q

$p^2 = 12$

(e) the number of possible positions for the point Q is greater than two, but finite

$p = 2\sqrt{3}$

Name: Claire Li   Date: 12.5

Math 9 Enriched: Section 3.5 Intersecting Lines

1. Find the intersection points for each set of lines algebraically:

a) $3x+y=3$ → $y=3-3x$

$2x+3y=-5$

$2x+3(3-3x)=-5$

$2x+9-9x=-5$

$-7x=-14$

$x=2$

$y=3-3x=3-6=-3$

∴ $\begin{cases} x=2 \\ y=-3 \end{cases}$

b) $3x-10y=16$ ①

$4x+2y=6$ ②

①+5②: $3x-10y+20x+10y=16+30$

$23x=46$

$x=2$

$3x-10y=16$

$6-10y=16$ ∴ $y=-1$

$\begin{cases} x=2 \\ y=-1 \end{cases}$

c) $6x-5y=\frac{4}{3}$ → $18x-15y=4$ ①

$10x+3y=6$ ②

①+5②: $18x-15y+50x+15y=4+30$

$68x=34$

$x=\frac{1}{2}$

$18x-15y=4$

$9-15y=4$

$5=15y$

$y=\frac{1}{3}$

∴ $\begin{cases} x=\frac{1}{2} \\ y=\frac{1}{3} \end{cases}$

d) $3x+7y=3$ ①

$4x-5y=42$ ②

$3②-4①$: $12x-15y-12x-28y=126-12$

$-43y=114$

$y=-\frac{114}{43}$

e) $\frac{x}{3}+\frac{y}{2}=\frac{1}{6}$ → $2x+3y=1$ ②

$x-6y=8$ → $x=8+6y$ ①

① → ②

$2(8+6y)+3y=1$

$16+12y+3y=1$

$15y=-15$

$y=-1$

$\begin{cases} x=2 \\ y=-1 \end{cases}$

f) $3x-10y=16$ ①

$4x+2y=6$ ②

①+5②: $3x-10y+20x+10y=16+30$

$23x=46$

$x=2$

$\begin{cases} x=2 \\ y=-1 \end{cases}$

g) $0.5x-0.\bar{6}y=6$ ①

$0.25x+0.\bar{3}y=-1$ ②

①+2②: $0.5x+0.5x=6-2$

$\begin{cases} x=4 \\ y=-6 \end{cases}$

h) $y+2x=10+4y$

$4(x+y)=42-y$

→ $\begin{cases} 4x-6y=20 & ① \\ 4x+5y=42 & ② \end{cases}$

②-①: $11y=22$

$y=2$

$\begin{cases} x=8 \\ y=2 \end{cases}$

i) $2(x-2y)=26-5y$

$3(y-x)=-2(y-7)$

→ $\begin{cases} 10x+5y=130 & ① \\ 5y-3x=14 & ② \end{cases}$

①-②: $13x=$

j) $2x-3y=10$ → $4x-6y=10$ ①

$-4x+6y=-30$ ②

k) $y=x^2$

$y=0.5x+5$

$x^2=0.5x+5$

$2x^2-x-10=0$

$(2x-5)(x+2)=0$

$x=\frac{5}{2}$ or -2

$\begin{cases} x=\frac{5}{2} \\ y=\frac{25}{4} \end{cases}$ or $\begin{cases} x=-2 \\ y=4 \end{cases}$

L) $y=\frac{4}{x}$

$1=3x-y$

$3x-1=\frac{4}{x}$

$3x^2-x-4=0$

$(3x-4)(x+1)=0$

$x=\frac{4}{3}$ or -1

$3x-1=y$

$\begin{cases} x=\frac{4}{3} \\ y=3 \end{cases}$ or $\begin{cases} x=-1 \\ y=-4 \end{cases}$

2. The lines with equations $px+3y=15$ and $6x+qy=30$ pass through the point (4,-3). What is the value of p+q?

(4,3)

$4p-9=15$

$4p=24$

$p=6$

$24-3q=30$

$3q=6$

$q=2$

$p+q=6+2=\boxed{8}$

3. The lines $x=\frac{1}{4}y+a$ and $y=\frac{1}{4}x+b$ intersect at point (1,2). What is a+b?

$1=\frac{1}{2}+a$

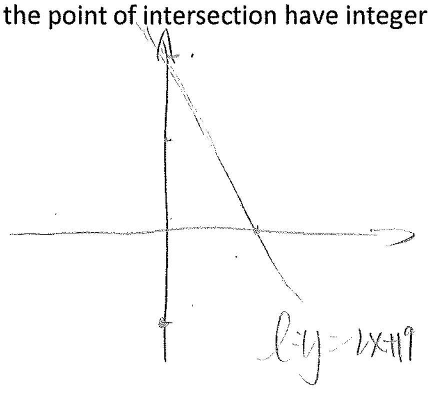
$a=\frac{1}{2}$

$2=\frac{1}{4}+b$

$b=\frac{7}{4}$

$a+b=\frac{1}{2}+\frac{7}{4}=\frac{9}{4}$

4. The line $-2x-y+19=0$ intersects the line $y=cx-10$ in the first quadrant. For how many values of "c" will the point of intersection have integer coordinates?



$-2x-y+19=0$

$y=-2x+19$

$l: y=cx-10$ has y int $(0,-10)$

$\begin{cases} y=-2x+19 \\ y=cx-10 \end{cases}$

$-2x+19=cx-10$

$(c+2)x=29$

∴ $c+2=1$ or 29

∴ Two values

5. If the point of intersection of the lines $x-3y=8$ and $2x-y=6$ is also on the line $kx+5y-12=0$, determine the value of *"k"*.

$$\begin{cases} x-3y=8 & ① \\ 2x-y=6 & ② \end{cases}$$

$2① - ②: 2x-6y-2x+y=16-6$

$-5y=10$

$y=-2$.

6. The lines y=ax+1 and y=1x+a, with "a" can not equal to 1, intersect in exactly one point. What is the x-coordinate of that point?

$$\begin{cases} y=ax+1 \\ y=x+a \end{cases}$$

$ax+1=x+a$

$(a-1)x=a-1$

$x=\boxed{1}$

7. Line are are concurrent if they pass through the same point. The lines $y=2x+3$ $y=8x+15$, and $y=5x+b$, are concurrent. What is the value of "b"?

$$\begin{cases} y=2x+3 \\ y=8x+15 \\ y=5x+b \end{cases}$$

$2x+3=8x+15$

$6x=-12$

$x=-2$.

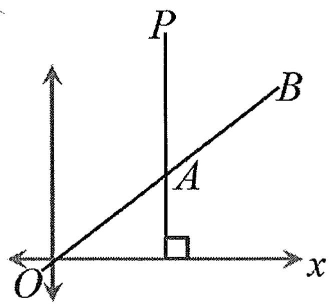
$y=-1$

$-1=5\times(-2)+b$

$-1=-10+b$

$\boxed{9}=b$

8. In the diagram, "A" lies on the line $x-2y=0$. Determine the length of *AP*.



9. The lines $y=mx+1$, where "m" is a positive integer, and $13x+9y=183$ intersect at a point P. What is the number of values of *"m"*, for which the coordinates of *"P"* are integers?

$174=2\cdot3\cdot29$.

$$\begin{cases} y=mx+1 \\ y=-\frac{13}{9}x+\frac{61}{3} \end{cases}$$

$mx+1=-\frac{13}{9}x+\frac{61}{3}$

$9mx+9=-13x+183$

$(9m+13)x=174$.

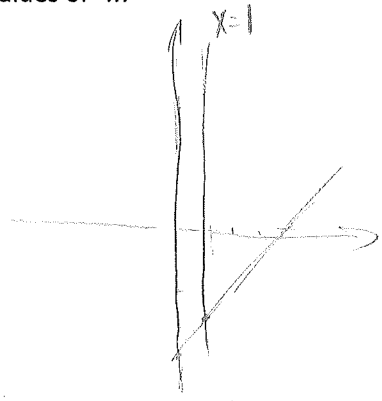
$58\times3=174$

$9m+13=58$

$9m=45$

$m=\boxed{5.}$

10. A triangle of area 9units^2 is formed by the x-axis and the lines *x = 1* and $y=mx-4$. Determine all possible values of *"m"*



X int : $\left(\frac{4}{m}, 0\right)$

$\frac{\left(\frac{4}{m}-1\right)\cdot(m-4)}{2}=9$

$\left(\frac{4}{m}-1\right)(4-m)=18$.

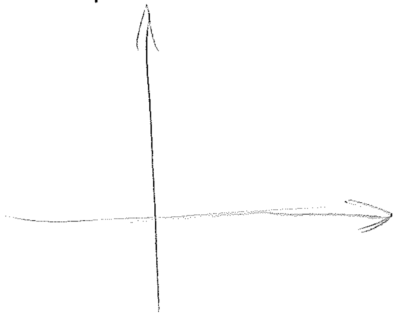
$\frac{16}{m}-4-4+m=18$

$\frac{16}{m}+m=26$.

$m^2-26m+16=0$.

$m=\boxed{13\pm3\sqrt{17}}$

11. A line through the point (-2,6) forms with the axes a triangle with area 25. Determine all possible values for the x-intercept of such a line.



$6 = -2m + b.$

$y\text{-int}\ (0, b).$

$6 - b = -2m$

$\frac{b-6}{2} = m$

$\therefore y = \frac{b-6}{2}x + b.$

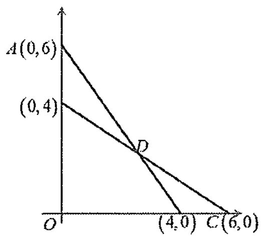
$x\text{-int}: \left(\frac{2b}{6-b}, 0\right)$

$\frac{2b}{6-b} \cdot b = 25.$

$2b^2 = 150 - 25b.$

$2b^2 + 25b - 150 = 0$

12. Calculate the area of the quadrilateral AOCD shown in the diagram.



$m_{AD} = \frac{-6}{4} = -\frac{3}{2}.$

$l_{AD}: y = -\frac{3}{2}x + 6.$

$m_{CD} = \frac{-4}{6} = -\frac{2}{3}.$

$l_{CD}: y = -\frac{2}{3}x + 4.$

$$\begin{cases} y = -\frac{3}{2}x + 6. \\ y = -\frac{2}{3}x + 4 \end{cases}$$

$-\frac{3}{2}x + 6 = -\frac{2}{3}x + 4$

$2 = -\frac{2}{3}x + \frac{3}{2}x.$

$12 = -4x + 9x.$

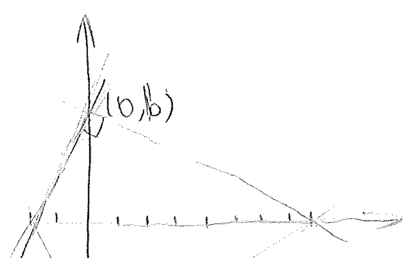
$x = \frac{12}{5} \therefore y = \frac{12}{5}$

$\therefore D\left(\frac{12}{5}, \frac{12}{5}\right).$

	0	6	
$\frac{72}{5}$	$\frac{12}{5}$	$\frac{12}{5}$	0
$\frac{72}{5}$	6	0	0
0	0	0	0
0	0	6	0
$\frac{144}{5}$			0

$\frac{\frac{144}{5} - 0}{2} = \boxed{\frac{12}{5}}$

13. Two perpendicular lines with x-intercepts -2 and 8 intersect at (0,b). Determine all values of "b".

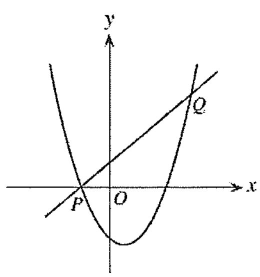


$\frac{b}{2} \cdot \frac{b}{-8} = -1$

$b^2 = 16.$

$b = \boxed{\pm 4}$

14. In the diagram, the line $y = x + 1$ intersects the parabola $y = x^2 - 3x - 4$ at the points P and Q. Determine the coordiantes of P and Q:



$$\begin{cases} y = x + 1 \\ y = x^2 - 3x - 4 \end{cases}$$

$x + 1 = x^2 - 3x - 4$

$0 = x^2 - 4x - 5$

$0 = (x-5)(x+1)$

$x = 5 \text{ or } -1$

$\therefore P(-1, 0).$

$Q(5, 6).$

15. If "a","b", and "c" are numbers such that the equations are all true, what is the value of "a + b – c"?

$$\begin{cases} -3b+7c = -10 & ① \\ b-2c = 3 & ② \end{cases}$$
$$a+2b-5c = 13$$

$①+3②: -3b+7c+3b-6c=-10+9$

$c=-1$

$\therefore b=1.$

$\Downarrow ③$

$a+2b-5c=13$

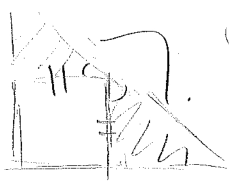
$a+2+5=13$

$a=6.$

$a+b-c=6+1+1=8.$

16. Through a point on the hypotenuse of a right triangle, lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangle is "m" times the area of the square. The ratio of the area of the other small right triangle to the area of the square is:

(A) $\frac{1}{2m+1}$ (B) m (C) $1-m$ (D) $\frac{1}{4m}$ (E) $\frac{1}{8m^2}$



congruent triangles.

17. Consider the systems of equation $x+y+z=a$, $xy+bz=6$. For what integer values of "a" and "b" does the system have infintely many integer solutions (x,y,z)? $\Downarrow$

$bz=6-xy$

$b=\frac{6-xy}{z}$

18. The solution to the pair of equations $4x-y+17=0$ and $3x+2y-1=0$ is also a solution to the equation $kx-2y+4=0$. What is the value of "k"?

$$\begin{cases} 4x-y=-17 & ① \\ 3x+2y=1 & ② \end{cases}$$

$2①+②: 8x-2y+3x+2y=-34+1$

$11x=-33$

$x=-3$

$\therefore y=5$

$kx-2y+4=0$

$-3k-10+4=0$

$-3k=6$

$k=\boxed{-2}$

Name: Claire **Date:** 12.10

Math 9/10 Honors: Section 3.6 Solving Problems with Linear Systems

1. Solve the following systems of equations:

a) $\frac{3}{4}x+\frac{13}{4}y=\frac{21}{4}$ $\qquad$ $\frac{1}{4}x+\frac{7}{4}y=\frac{11}{4}$

$\hookrightarrow 3x+13y=21$ ① $\qquad$ $\hookrightarrow x+7y=11$ ②

3② − ①: $21y-13y=33-21$

$8y=12$

$y=\frac{3}{2}$

$\therefore \begin{cases} x=\frac{1}{2} \\ y=\frac{3}{2} \end{cases}$

b) $75x+270y=54$ $\qquad$ $25x+90y=18$

$\hookrightarrow 25x+90y=18$

∞ solutions.

c) $\frac{5}{3}x+\frac{7}{3}y=37.50$ $\qquad$ $\frac{8}{3}x+\frac{4}{5}y=16$

$5x+7y=112.50$ $\qquad$ $10x+3y=60$ ②

2① − ②: $10x+14y-\cancel{10x}-3y=225-60$

$11y=165$

$y=15$

$\therefore \begin{cases} x=1.5 \\ y=15 \end{cases}$

d) $\frac{8}{3}x+\frac{5}{2}y=-\frac{56}{50}$ $\qquad$ $\frac{7}{3}x+\frac{7}{4}y=-0.98$

$400x+375y=-16.8$ ① $\qquad$ $4x+3y=-1.68$ ②

2. Indicate the number of solutions for each system:

a) $6x+10y=24$ $\qquad$ $3x+5y=12$

$6x+10y=24$

Infinite solutions.

b) $19y-14x=26$ ① $\qquad$ $7x-9y=13$

$14x-18y=26$ ②

① + ②: $19y-18y=52$

$y=52$

$\therefore \begin{cases} x= \\ y=52 \end{cases}$

c) $\frac{2}{3}x+\frac{8}{3}y=12$ $\qquad$ $6x+18y=72$

$x+4y=18$ ① $\qquad$ $x+3y=12$ ②

① − ②: $4y-3y=18-12$

$y=6$

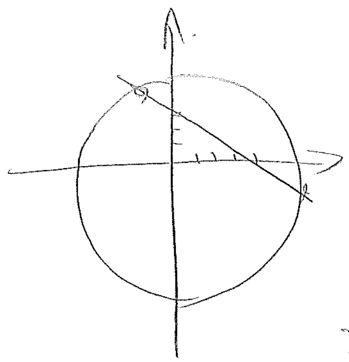
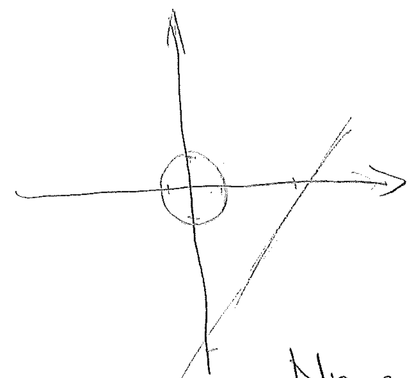
$\therefore \begin{cases} x=-6 \\ y=6 \end{cases}$

One solution

d) $10x-8y=15$ $\times 2$ $\qquad$ $-4x+3.2y=-6$ $\times 5$

$20x-16y=30 \longleftrightarrow 20x-16y=30$ ②

Infinite solutions.

e) $x^2+y^2=25$ $\quad 2x+3y=4$ $3y=4-2x$ $y=-\frac{2}{3}x+\frac{4}{3}$  2 solutions	f) $y^2+x^2=1$ $\quad 4x+3y=-24$ $3y=-24-4x$ $y=-8-\frac{4}{3}x$  $y^2=1-x^2$ $y=\sqrt{(1-x)(1+x)}$ No solution

3. Two lines have no solution. The equation of the first line is $x-\sqrt{3}y+2\sqrt{3}=0$. What is the slope of the 2nd line?

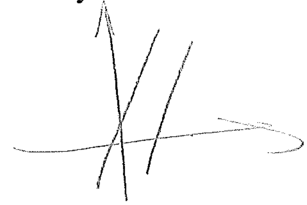
$-\sqrt{3}y=-2\sqrt{3}-x$

$y=2+\frac{x}{\sqrt{3}}$

$m=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

4. When two lines are inconsistent, how many solutions would they have? What does it mean that two lines are inconsistent?

0 solutions.



5. Word Problems: The first number is twice the second. The sum of five times the first and 1.5 times the second is equal to 17.25. Find the numbers

$2x \quad x$

$5\times2x+1.5x=17.25$

$11.5x=17.25$

$x=\frac{3}{2}$

$\boxed{\frac{3}{2} \text{ and } 3}$

6. The first number is 8 times smaller than the second. The sum of four times the first and triple the second is 14. Find the numbers. the second # = a

$4\cdot\frac{a}{8}+3a=14$

$\frac{a}{2}+3a=14$

$7a=28$

$a=4$

$\boxed{\frac{1}{2} \text{ and } 4}$

7. The second number is two less than the first. The sum of half the first number is one third the second is equal to 8.5. Find the numbers.

the second # = a

$\frac{a-2}{3}+\frac{a}{2}=\frac{17}{2}$

$3a+2(a-2)=51$

$5a=55$

$a=11$

$\boxed{11 \text{ and } 9}$

8. Under what conditions will the system have no solution?

$mx+ny=p$

$vx+wy=z$

$\boxed{\frac{m}{v}=\frac{n}{w}\neq\frac{p}{z}}$

9. If $3x - 2y - z = 0$ and $x + y - z = 0$, find the ratio of x:y.

$3x-2y-\cancel{z} = x+y-\cancel{z}$

$2x = 3y$

$x:y = \boxed{2:3}$ //

$\frac{x}{y} = \frac{2}{3}$

10. If the system of equations: $2x + 4y = 3k$ and $x + 2y = 3$ has atleast one solution, then "k" can not be what value?

×2.

$3k \neq 3$

$k \neq \boxed{1}$ //

11. If $2x + y = -3$ and $2y - x = 4$, then what is the value of $x + y$?

$\begin{cases} 2x+y=-3 & ① \\ -2x+4y=8 & ② \end{cases}$

①+②: $5y=5$

$y=1$

$\therefore x=-2$

$x+y=-1$

12. If $(1,2)$ is a solution to the system $ax + by = 1$ and $bx + ay = 2$, then what is the value of $a - b$?

$\begin{cases} a+2b=1 & ① \\ b+2a=2 & ② \end{cases}$

②-①: $a-b=\boxed{1}$ //

13. If the straight lines $2x + hy = 11$ and $kx + 4y = 8$ intersect at point $(4,-1)$, then what are the values of "h" and "k?

(4, -1)

$\begin{cases} 2x+hy=11 \\ kx+4y=8 \end{cases}$

$8-h=11$

$h=-3$

$4k-4=8$

$k=3$

$\begin{cases} k=3 \\ h=-3 \end{cases}$

14. Given the two systems of equations: $\begin{matrix} ax + by = c \\ dx + ey = f \end{matrix}$, indicate what happens under each scenario:

a) $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ — ∞ solutions.

b) $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$ — 0 solutions.

c) $\frac{a}{d} = \frac{c}{f} \neq \frac{b}{e}$ — 1 solution

d) $ae = bd, c = 0$ — 0 solution.

15. Andy invested $13000 into two different Mutual Fund. The first fund has a return of 3%, and the other at 5%. If the total interest earned in one year is $530, how much did he invest in each of the mutual funds?

first fund amount = x.

second fund. amount = y.

$\begin{cases} x+y=13000 & ① \\ \frac{3x}{100}+\frac{y}{20}=530 & ② \end{cases}$

100② - 3①: $5y-3y=14000$

$2y=14000$

$y=7000$

$\therefore x=6000$

16. Barry borrowed $155,000 to mortgage his apartment from two different banks. BMO charged an interest of 20%, and Royal Bank at 18.75%. If the amount of interest he paid to both banks in one was equal, how much did he borrow from each bank?

Amount to BMO = x

Amount to RBC = y.

$\begin{cases} x+y=155000 \; ① \\ \frac{1}{5}x=\frac{3}{16}y \; ② \end{cases}$

②: $y=\frac{16}{15}x$.

② → ①: $x+\frac{16}{15}x=155000$

$\frac{31}{15}x=155000$

$x=7500$

$\therefore y=80000$

$\begin{cases} \text{BMO } \$7500. \\ \text{RBC } \$80000. \end{cases}$

17. A rock concert at GM place sold 17,500 seats. There are level A seats that cost $20/ticket and level B seats that cost $50. If total revenue was $425000, how many of each seat did they sell?

# of A = x

# of B = y.

$\begin{cases} x+y=17500 \; ① \\ 20x+50y=425000 \; ② \end{cases}$

② − 20①: $30y=75000$

$y=2500$.

$\therefore x=15000$.

$\begin{cases} \text{15000 A seats.} \\ \text{2500 B seats} \end{cases}$

18. Six hotdogs and 10 burgers cost $40. 3 Hotdogs and 8 burgers cost $29.75. How much does each burger cost?

~~Cost of~~ hotdog : x

Cost of burger = y.

$\begin{cases} 6x+10y=40 \; ① \\ 3x+8y=29.75 \; ② \end{cases}$

2② − ①: $16y-10y=59.5-40$.

$6y=19.5$

$y=3.25$.

$\boxed{\$3.25}$ //

19. Jerry is a stock broker and he own 1000 shares of Starbucks stock. Two months later, he own 1500 shares but the value of each share depreciated to 60% of its original value. Furthermore, the total value of his investments depreciated by $5000. How much was each share worth before the depreciation?

Share worth before depreciation = x.

$1000x-1500\times\frac{3}{5}x=5000$

$1000x-900x=5000$

$x=50$.

$\boxed{\$50}$ //

20. Lucy is taking professional dancing lessons. 9 lessons cost $725 and 20 lessons will cost her $1550. If there is a one time administration fee, how much does each lesson cost?

admin fee = x.

class cost = y

$\begin{cases} x+9y=725 \; ① \\ x+20y=1550 \; ② \end{cases}$

② − ①: $11y=825$

$y=75$

Each lesson cost $75.

21. In a hockey league, players are awarded from point system where each goal is worth 2 points and assists are worth 1 point. Last season, James earned 130 points. However, if the point system was switched (where goals are worth 1 point and assists 2 points) he would have 110 points. How many goals and assists does he have?

# of goals. = x

# of assists = y.

$\begin{cases} 2x+y=130 \; ① \\ x+2y=110 \; ② \end{cases}$

2② − ①: $4y-y=220-130$

$3y=90$. $y=30$.

$\therefore x=50$.

$\boxed{\text{50 goals. 30 assists}}$ //

22. Given that 0 < x < y < 100, what is the number of integer solutions (x,y) to the equation 2x + 3y = 50 :

$\begin{cases} x= 1, \; 4, \; 7, \; \cancel{10} \\ y= 16, \; 14, \; 12, \; \cancel{10} \end{cases}$

$\boxed{3}$ //

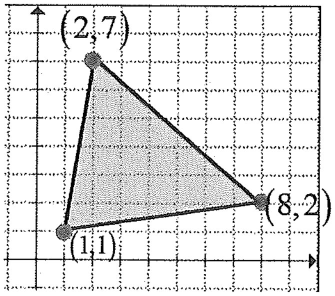
Name: Claire

Date: 12.12.

Math 9/10 Honours: Section 3.7 Shoe Lace Method

1. Given the vertices, find the area of each 2D Shape using the Shoe Lace Method. Show all your work & steps:

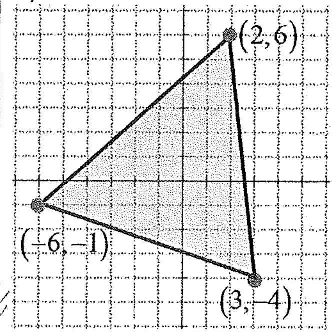
a)



2 7
7 1 1 2
26 8 2 2
4 2 7 56
37 60

$S = \frac{60-37}{2} = \boxed{11.5}$

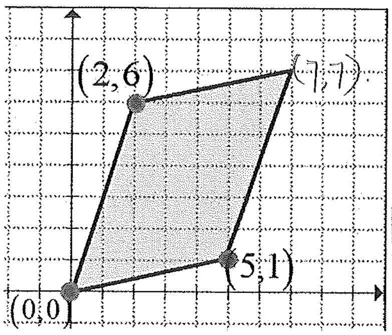
b)



2 6
−36 −6 −1 −2
−3 3 −4 24
−8 −2 6 18
−47 40

$S = \frac{40-(-47)}{2} = \boxed{43.5}$

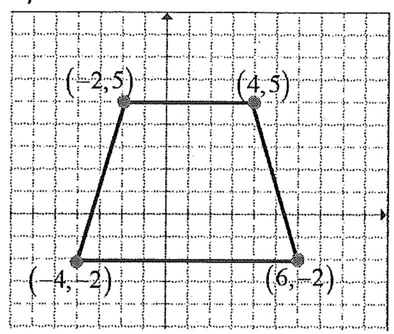
c)



2 6
0 0 0 0
0 5 1 0
7 7 7 35
14 2 6 42
21 77

$S = \frac{77}{21} = \boxed{28}$

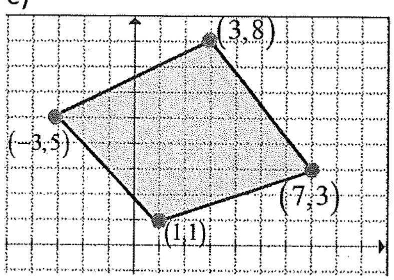
d)



−2 5
20 4 5 −10
30 6 −2 −8
8 −4 −2 −12
4 −2 5 −20
62 −60

$S = \frac{62-(-60)}{2} = \boxed{61}$

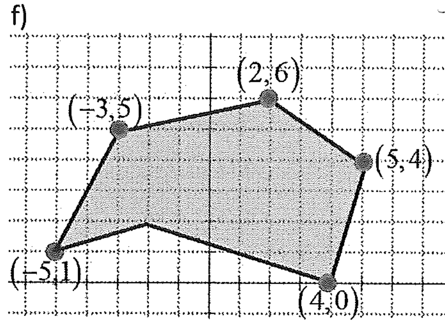
e)



3 8
56 7 3 9
3 1 1 7
−3 −3 5 5
15 3 8 −24
71 −3

$S = \frac{71-(-3)}{2} = \boxed{37}$

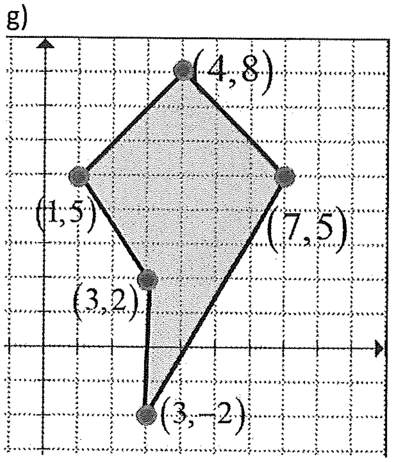
f)



2 6
30 5 4 8
16 4 0 0
0 −5 1 4
−3 −3 5 −25
10 2 6 −18
53 −31

$S = \frac{53-(-31)}{2} = \boxed{42}$

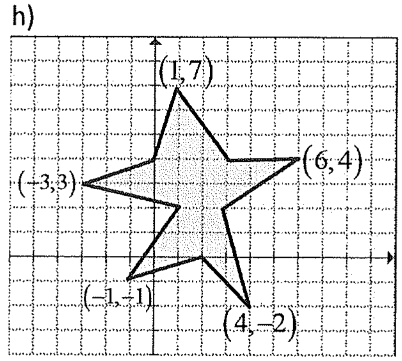
g)



4 8
56 7 5 40
15 3 −2 −14
−6 3 2 6
2 1 5 15
20 4 8 8
87 55

$S = \frac{87-55}{2} = \frac{32}{2} = \boxed{16}$

h)

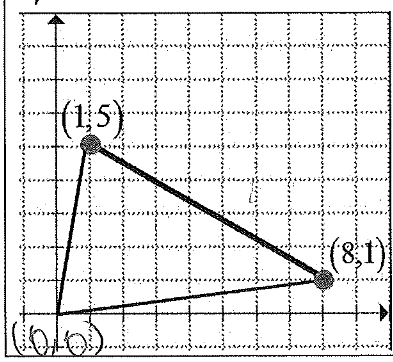


1 7
42 6 4 4
16 4 −2 −12
2 −1 −1 −4
3 −3 3 −3
3 1 7 −21
66 −36

$S = \frac{66-(-36)}{2} = \boxed{51}$

2. A triangle is created using the origin (0,0) and the coordinates of the endpoints of the given line. Use the determinant to find the area of that triangle:

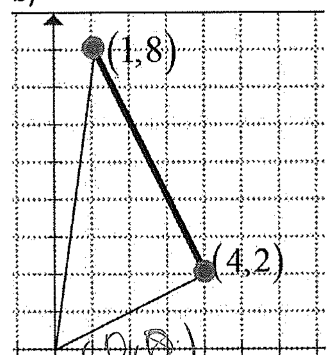
a)



1 5
40 8 1 1
0 6 0 0
0 1 5 0
40 1

$\frac{40-1}{2} = \boxed{\frac{39}{2}}$

b)



1 8
32 4 2 2
0 0 0 0
0 1 8 0
32 2

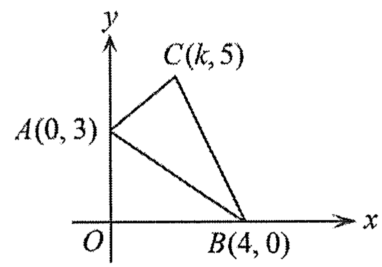
$\frac{32-2}{2} = \boxed{15}$

3. Using the standard xy-coordinate plane, what is the area (in square units) of a triangle whose vertices have the coordinates (0,0) , (1,5), and (7,3)?

0 0
0 1 5 0
35 7 3 3
0 0 0 0
35 3

$\frac{35-3}{2} = \frac{32}{2} = \boxed{16}$

4. A triangle has vertices $A(0,3)$, $B(4,0)$, *and* $C(k,5)$, where $0 < k < 4$. If the area of the triangle is 8, determine the value of "k":



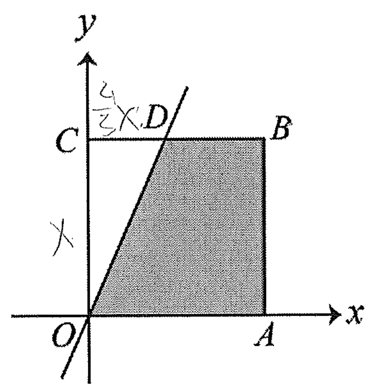
k 5
20 4 0 0
0 0 3 12
3k k 5 0
3k+20 12

$\frac{3k+20-12}{2} = 8$

$3k+8 = 16$

$k = \boxed{\frac{8}{3}}$

5. In the diagram, OABC is a square. A line $y = mx$ intersects CB at point "D" and cuts out 1/3 of the area of the square. What is the value of "m"?



Let OA = OC = BC = BA = X.

$S_{\triangle OCD} = \frac{1}{3}x^2 = \frac{OC \cdot CD}{2}$

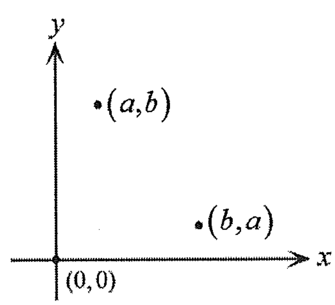
$\therefore CD = \frac{2}{3}X$

$\therefore D(\frac{2}{3}X, X)$

$X = \frac{2}{3}X \cdot m$

$m = \frac{3}{2}$

6. The three vertices of a triangle are given in the diagram. If the area of triangle is 20 units squared, then what is the value of a+b?



a b
b^2 b a a^2
0 0 0 0
b a b 0
b^2 a^2

$\frac{a^2-b^2}{2} = 20$

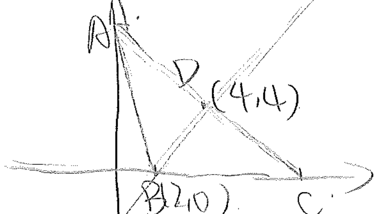
$a^2-b^2 = 40$

$(a+b)(a-b) = 40$

a+b = 1, 2, 4, 5, 8, 10, 20, 40.

7. Triangle ABC has vertices A(0,8), B(2,0), and C(8,0).

a) Determine the equation of the line through "B" that cuts the triangle in half



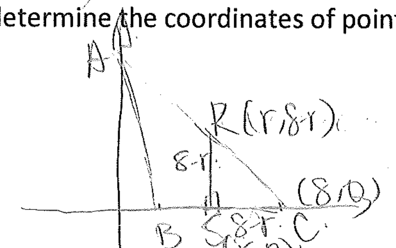
Midpoint of AC: D(4,4)

$m_{BD} = \frac{4}{2} = 2$

$\therefore \ell_{BD}: y = 2x - 4$

b) A vertical line intersects AC at "R" and BC at "S", forming triangle RSC. If the area of triangle RSC is 12.5, determine the coordinates of point "R"

Let $R(r, 8-r)$. $\therefore \boxed{R(3,5)}$



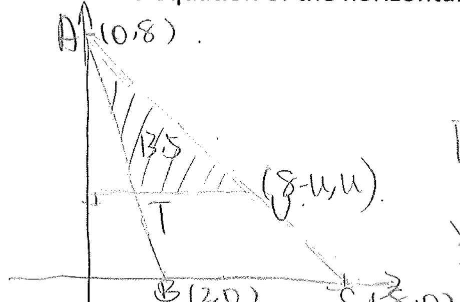
$\frac{RS \cdot SC}{2} = 12\frac{1}{2}$

$RS^2 = 25$

$RS = 5$

c) A horizontal line intersects AB at "T" and AC at "U", forming triangle ATU. If the area of triangle ATU is 13.5, determine the equation of the horizontal line.

Let $U(8-u, u)$.



$\left(\frac{AU}{AC}\right)^2 = \frac{S_{\triangle ATU}}{S_{\triangle ABC}}$

$\frac{[\sqrt{2}(8-u)]^2}{128} = \frac{13.5}{24}$

$\frac{\cancel{2}(8-u)^2}{\cancel{128}\,64} = \frac{9}{16}$

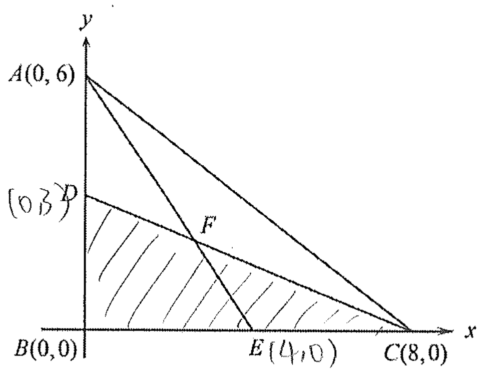
$(8-u)^2 = 6^2$

$8 - u = 6 \ (\cancel{-6})$

$u = 2$

$\therefore \boxed{y = 2}$

8. In the diagram, "D" and "E" are midpoints of AB and BC respectively. Use the diagram to answer the following questions:



a. Determine an equation of the line through "C" and "D"

$m_{CD} = \frac{-3}{8} = -\frac{3}{8}$

$\therefore \ell_{CD}: y = -\frac{3}{8}x + 3$

b. Determine the coordinates of "F", which is the intersection of AE and CD

$m_{AE} = \frac{-6}{4} = -\frac{3}{2}$

$\therefore \ell_{AE}: y = -\frac{3}{2}x + 6$

$\begin{cases} \ell_{AE}: y = -\frac{3}{2}x + 6 \\ \ell_{CD}: y = -\frac{3}{8}x + 3 \end{cases}$

$-\frac{3}{2}x + 6 = -\frac{3}{8}x + 3$

$-12x + 48 = -3x + 24$

$-9x = -24$

$x = \frac{8}{3}$

$\therefore F\left(\frac{8}{3}, 2\right)$

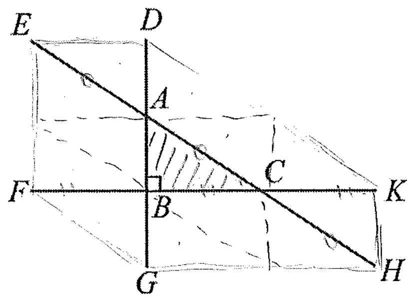
c. Determine the area of triangle DBC

$\frac{DB \cdot BC}{2} = \frac{3 \cdot 8}{2} = \boxed{12}$

d. Determine the area of quadrilateral DBEF:

$$\frac{16-0}{2} = \boxed{8}$$

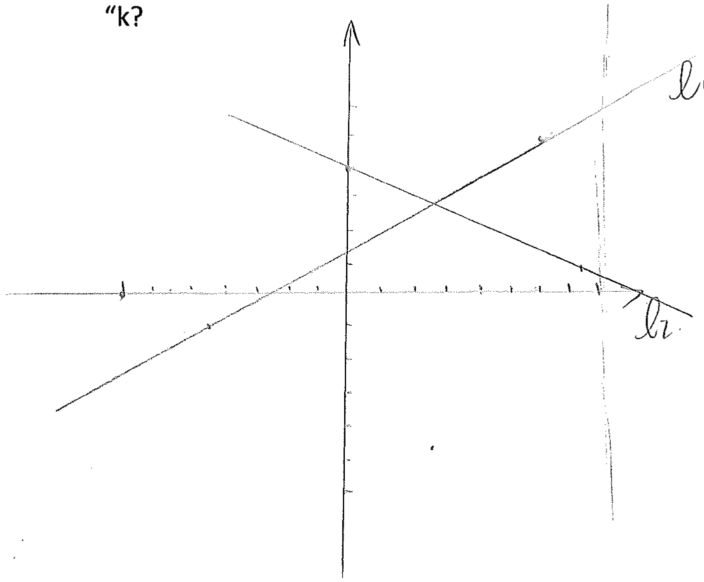
9. In the diagram, ΔABC is right-angled. Side AB is extended in each direction to points "D" and "G" such that $DA = AB = BG$. Similarly, BC is extended to points "F" and "K" so that $FB = BC = CK$, and AC is extended to points "E" and "H" so that $EA = AC = CH$. What is the ratio of the area of hexagon DEFGHK to the area of ΔABC?



$S_{EFAB} = S_{BGHC} = S_{ACKD} = 3S_{ABC}$.

$\therefore \dfrac{S_{DEFGHK}}{S_{ABC}} = \dfrac{3\times3+3+1}{1} = \boxed{13}$

10. Challenge: There are 3 lines. The first line goes through points (-9,-2) and (12,10). The second line goes through points (0,8) and (15,2). The third line goes through points (-14,0) and (16,k). The three lines intersect and form a triangle in the middle. If the area of the triangle is 17 units^2, then what is the value of "k?



$m_1 = \dfrac{10+2}{12+9} = \dfrac{12}{21} = \dfrac{4}{7}$

$\therefore l_1: y = \frac{4}{7}x + \frac{22}{7}$

$m_2 = \dfrac{-6}{15} = -\dfrac{2}{5}$

$\therefore l_2: y = -\frac{2}{5}x + 8$

$m_3 = \dfrac{k}{30}$

$\therefore l_3: y = \frac{k}{30}x + \frac{7k}{15}$

$l_1 \& l_2 = (5, 6)$

$l_1 \& l_3: \left(\dfrac{98k-660}{7k-120},\right.$